LESSON 1.4

Variables and Constants

You will need:

the Lab Gear 

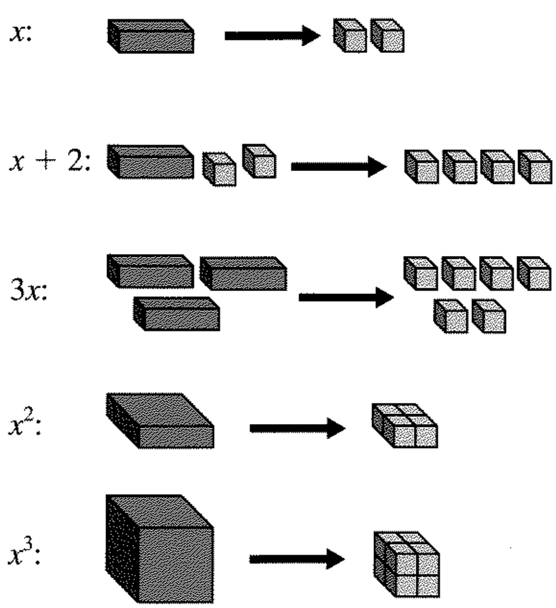
Variables are one of the most important concepts of algebra. A variable can stand for different numbers at different times. For example, x could be a positive or a negative number or 0. It could be greater than y, less than y, or equal to y.

Because they do not change, numbers are called *constants.*

SUBSTITUTING

Definition: Replacing a variable by a constant amount is called *substitution.*

Example: The figure shows how the Lab Gear can be used to show the substitution $x = 2$ for the expressions x, $x + 2$, $3x$, x^2, and x^3.



1. Sketch what $x + 2$ looks like modeled with the Lab Gear. Then sketch what it looks like if the following substitution is done.
   a. $x = 5$ b. $x = 1$
   c. $x = 3$ d. $x = 0$
2. Repeat problem 1 for x^2.
3. Repeat problem 1 for x^3.
4. Repeat problem 1 for $3x$.

An expression that involves x can have many different values, depending on the value of x.

EVALUATING

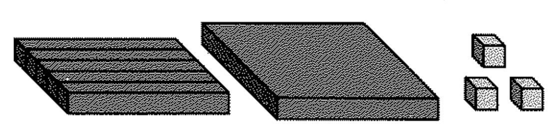
Definition: To *evaluate* an expression means to find its value for a particular value of x.

Looking back at the figure in the previous section, you can see the value of each expression when $x = 2$. The figure shows that $x + 2 = 4$, $3x = 6$, $x^2 = 4$, and $x^3 = 8$.

In the following problems:

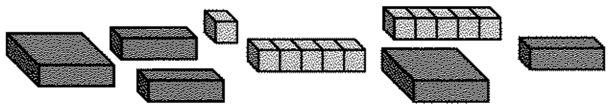
- Put out blocks to match each figure.
- Replace the variables (represented by blue blocks) with the given constants (represented by yellow blocks).
- Evaluate each expression by counting what you have.

5. Evaluate for :
   a. $y = 1$; b. $y = 2$; c. $y = 0$.



6. Evaluate for :

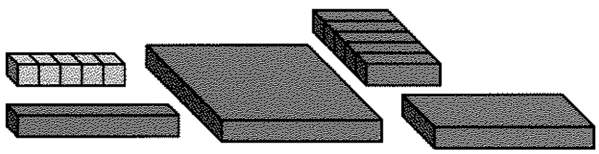
a. $x = 1$; b. $x = 5$; c. $x = 0$.



7. Evaluate for :

a. $x = 5$ and $y = 4$

b. $x = 4$ and $y = 0$



Evaluate these expressions without using the Lab Gear. You may want to use your calculator.

8. $y^2 + 5y + 3$ if $y = 1.3$

9. $y^2 + xy + 5x + y + 5$ if $x = \frac{1}{2}$ and $y = 4$

Evaluating expressions is important in many walks of life, from science and engineering to business and finance. It is usually done with the help of calculators and computers. In this course you will learn some of the ideas that are built into calculators and computers.

FINDING X

Use trial and error for these problems.

10. If $x + 2 = 18$, what is x?

11. What is x if

a. $3x = 18$? b. $x^2 = 64$?

c. $x^3 = 64$?

In a sense, finding x is the reverse of substituting. In future chapters you will learn many methods for finding the value of a variable.

THE SUBSTITUTION RULE

In the following equations, there are two place-holders, a diamond and a triangle. The **substitution rule** is that, within one expression or equation, the same number is placed in all the diamonds, and the same number is placed in all the triangles. (The number in the diamonds may or may not equal the number in the triangles.)

For example, in the equation

$$\Diamond + \Diamond + \Diamond + \Delta = \Delta + \Delta$$

if you place 2 in the $\Diamond$ and 3 in the Δ, you get

$$2 + 2 + 2 + 3 = 3 + 3.$$

Note that even though the diamond and triangle were replaced according to the rule, the resulting equation is *not* true.

12. Exploration The equation

$$\Diamond + \Diamond + \Diamond + \Delta = \Delta + \Delta$$

is not true with 2 in the $\Diamond$ and 3 in the Δ. Find as many pairs of numbers as possible that can be put in the $\Diamond$ and in the Δ to make the equation true. For example, 0 in both the Δ and $\Diamond$ makes it true. Arrange your answers in a table like this:

$\Diamond$	Δ
0	0
...	...

Describe any pattern you notice. Explain why the pattern holds.

For the following equations, experiment with various numbers for ◊ and Δ. (Remember the substitution rule.) For each equation, try to give three examples of values that make it true. If you can give only one, or none, explain why.

13. $\Diamond + \Diamond + \Diamond = 3 \cdot \Diamond$

14. $\Diamond + \Diamond + \Diamond = 4 \cdot \Diamond$

15. $\Delta + \Delta + \Delta = 3 \cdot \Delta$

16. $\Diamond + \Diamond + 2 = 3 \cdot \Diamond$

17. $\Diamond + \Diamond + 2 = 2 \cdot \Diamond$

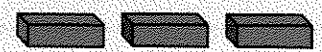
18. $\Diamond \cdot \Delta = \Delta \cdot \Diamond$

19. $\Diamond \cdot \Delta = \Delta + \Diamond$

20. $\Diamond \cdot \Diamond \cdot \Diamond = 3 \cdot \Diamond$

21. $\Diamond \cdot \Diamond \cdot \Delta = \Diamond + \Diamond + \Delta$

22. **Report** Say that ◊ is x and Δ is y. For each equation above, show both sides with a sketch of Lab Gear blocks. In some cases, the sketches may help you explain whether the equations are always true or not. For example, for problem 13 both sides would look like this.



But, for problem 14 the right side would look like this. Write an illustrated report about what you did.