LESSON 6.7

How Much More Than? How Many Times as Much?

You will need:

graph paper

the Lab Gear

COMPARING AGES

On Mark's 12th birthday, he said to his little brother Gordon, "You'd better do what I say. Now I'm twice as old as you are."

The six-year-old math whiz wasn't scared. "That's nothing," he laughed. "A few years ago, you were four times as old as I was. And not long after I was born, you were *thirty-seven* times as old as I was."

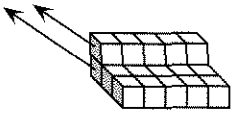
1. How old were the two brothers when
   a. Mark was four times as old as Gordon?
   b. Mark was 37 times as old?
2. a. As Mark and Gordon get older, does the *difference* between their ages increase, decrease, or stay the same? Explain.
   b. Does the *ratio* of their ages increase, decrease, or stay the same? Explain.
3. Mark was born in 1980. On the same axes, make two graphs, one showing Mark's age as a function of time and the other showing Gordon's age as a function of time. Label the x-axis *years after 1980* and the y-axis *age*. Compare the two graphs.
4. a. Make a graph showing the *difference* between the two boys' ages as a function of time. Label the x-axis *years after 1980* and the y-axis *difference in ages*. Describe your graph.
   b. Make a graph showing the *ratio* of Mark's age to Gordon's age as a function of time. Label the x-axis *years after 1980* and the y-axis *ratio of ages*. Describe your graph.
   c. Compare the two graphs.
5.
   a. Why do we usually compare people's ages using differences instead of ratios?
   b. What do you think is the smallest possible value for this ratio of Mark's age to Gordon's age? Explain.
6. Beau and Bea said, "The ratio of our ages will always be the same!" How could this be? Discuss.
7. On Mark's 12th birthday, his mother was three times as old as Mark. Was she ever twice as old? Was she ever four times as old? Explain.

COMPARING NUMBERS

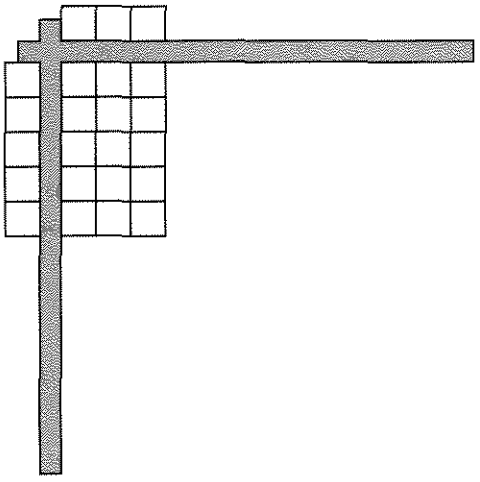
When comparing the size of two positive numbers, for example 5 and 15, you can ask two different questions.

- 15 is *how much more than* 5?
- 15 is *how many times as much* as 5?

The question *How much more than...?* is answered using subtraction, as shown in this figure. Since $15 - 5 = 10$, you can say that 15 is 10 more than 5, (or 10 is the difference of 15 and 5).



The question *How many times as much...?* is answered using division, as shown with the Lab Gear. Since 15/5 = 3, 15 is 3 times as much as 5, (or 3 is the ratio of 15 and 5).



Answer both questions about these pairs of numbers in problems 8-13. Show how you got your answers. In some cases, you may want to use the Lab Gear.

a. The first number is *how much more than* the second?

b. The first number is *how many times as much* as the second?

8. 35 and 5

9. 10 and 10

10. 9 and 8

11. 16 and 4

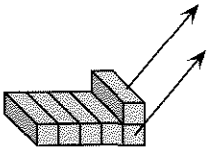
12. 16 and $\frac{1}{4}$

13. 4 and 16

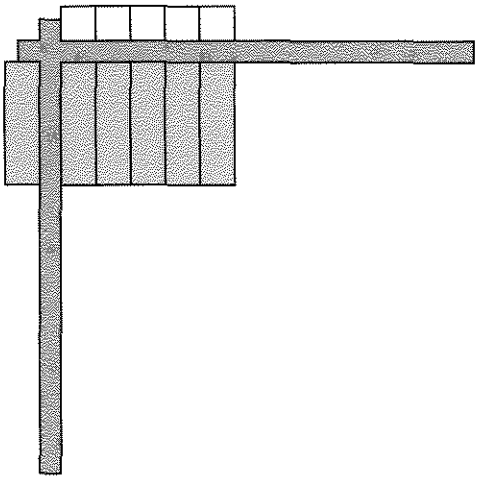
COMPARING ALGEBRAIC EXPRESSIONS

Sometimes you need to compare quantities given by formulas that involve variables. The same methods can be used as when comparing numbers.

To find out *how much more* $5x$ is than x, subtract $5x - x$, as shown.



To find out *how many times as much* $5x$ is than x, divide as shown.



For each pair of expressions in problems 14-18:

a. The first expression is *how much more than* the second?

b. The first expression is *how many times as much* as the second?

14. $5x$ and x

15. $10x$ and 5

16. $10x$ and $5x$

17. $8xy$ and $2x$

18. $2x + 2y$ and $x + y$

APPLICATIONS

19. The Statue of Liberty, which guards the entrance to New York harbor, was given to the United States by the people of France in honor of the centennial of American independence. The statue measures 111 feet 1 inch, from her heel to the top of her head. She was designed by Frederic Auguste Bartholdi. Suppose Mr. Bartholdi had used as a model for the statue a woman who was 5 feet 1 inch tall.

a. How much taller is the statue than the model?

b. How many times as tall is the statue?

c. Which of these two numbers would have been useful to Mr. Bartholdi when designing the statue? Explain.

20. If Reg takes the bus to work, it takes him about an hour and 15 minutes. If he drives, it takes him about 45 minutes.

a. How much longer does it take on the bus?

b. How many times as long does it take?

c. Which number would be more important to Reg in deciding which method of transportation to use? Why?

21. The A.R. Bagel Company charged 30 cents for a bagel in 1973 and 60 cents in 1983. During the same period of time, the hourly wage of a bagel deliverer increased from $2.50 per hour to $5.00 per hour. The company president said, "We try to pay our employees the highest possible wages and charge our customers the lowest possible prices. In a period of high inflation, our prices have risen only 30 cents in ten years. Yet, during the same time, we doubled hourly wages." How might the president of the Bagel Workers' Union describe this situation? Discuss.

DISCOVERY *TOURNAMENTS*

22. Twelve teams are playing in a tournament.

a. Each team must be scheduled to play three games with each other team. How many games must be scheduled? (Hint: Start by thinking of a smaller tournament.)

b. The teams play "best out of three" games. In other words, the third game of the three may not get played. What is the smallest number of games that might be played?