LESSON 8.10

Working With Monomials

The product of the monomials $3x^2$ and $9x^4$ is also a monomial. This can be shown by using the definition of exponentiation as repeated multiplication.

$$3x^2 = 3 \cdot x \cdot x \text{ and } 9x^4 = 9 \cdot x \cdot x \cdot x \cdot x$$

so

$$3x^2 \cdot 9x^4 = 3 \cdot x \cdot x \cdot 9 \cdot x \cdot x \cdot x \cdot x = 27x^6$$

1. Find another pair of monomials whose product is $27x^6$.

2. **Exploration** If possible, find at least two answers to each of these problems. Write $27x^6$ as:

a. the product of three monomials

b. the sum of three monomials

c. a monomial raised to a power

d. the quotient of two monomials

e. the difference of two monomials

PRODUCT OF POWERS

The monomial $48x^9$ can be written as a product in many different ways. For example, $16x^6 \cdot 3x^3$ and $12x^5 \cdot 4x^3 \cdot x$ are both equal to $48x^9$.

3. Write $48x^9$ in three more ways as a product of two or more monomials.

4. Write $35x^4$ as a product in which one of the factors is

a. a third-degree monomial;

b. a monomial with a coefficient of 7;

c. $5x^0$;

d. $35x^3$.

5. Write $7.2 \cdot 10^8$ in three ways as the product of two numbers in scientific notation.

6. Write x^5 in three ways as a product of two or more monomials.

7. **Generalization** Study your answers to problem 6. Then fill in the exponent.

$$x^a \cdot x^b = x^?$$

Explain.

8. If possible, write each expression more simply. If it is not possible, explain why not.

a. $3x^5 \cdot 6x^4$ b. $x^5 \cdot y^7$

c. $y^7 \cdot y^3$ d. $4a^4 \cdot 9a^3$

The generalization you made is one of the laws of exponents. It is sometimes called the *product of powers* law. It says that

$x^a \cdot x^b = x^{a+b}$, as long as x is not 0.

However, notice that it works only when the bases are the same.

POWER OF A PRODUCT

The expression $x^4 \cdot y^4 \cdot z^4$ is the product of three powers. Since the bases are not the same, we cannot use the product of powers law. However, notice that since the exponents are the same, it is possible to write a product of powers as a single power

$$x^4y^4z^4 = x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z$$
$$= xyz \cdot xyz \cdot xyz \cdot xyz$$
$$= (xyz)^4$$

9. Write $16a^2b^2$ as the square of a monomial. (Hint: First rewrite 16 as a power.)

10. Write p^3q^3 as the cube of a monomial.

11. If possible, write each expression as a single power. If it is not possible, explain why not.

a. $32n^5m^5$ b. x^2y^3

c. $(2n)^7 \cdot (3m)^7$ d. $(ab)^4 \cdot (bc)^4$

The generalization you used above is another of the laws of exponents. It is sometimes called the *power of a product* law. It says that

$x^a y^a = (xy)^a$, as long as x and y are not 0.

However, notice that it works only when the exponents are the same.

12. Write without parentheses.

a. $(6y)^2$ b. $(3xy)^4$

c. $(5xyz)^3$ d. $(2x)^3$

e. $(2xy)^3$ f. $(2xyz)^3$

13. Write $64x^3y^6z^9$ as the cube of a monomial.

POWER OF A RATIO

14. Write 49/25 as the square of a ratio.

Study this example.

$$\left(\frac{x}{y}\right)^3 = \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} = \frac{x \cdot x \cdot x}{y \cdot y \cdot y} = \frac{x^3}{y^3}$$

This law of exponents is called the *power of a ratio* law. It says that

$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$, as long as x and y are not 0.

However, notice that it works only when the exponents are the same.

15. Write as a power of a ratio.

a. $8x^3/y^6$ b. $16x^4/x^{10}$

16. Write as a ratio of monomials.

a. $(5x/7z)^9$ b. $(2xy/yz)^2$

RATIOS OF MONOMIALS

Consider the ratio $6x^5/4x^7$. By multiplying numerator and denominator by x, you can get the equivalent ratio $6x^6/4x^7$. Or you can get an equivalent ratio in lowest terms by noticing that

$$\frac{6x^5}{4x^7} = \frac{3}{2x^2} \cdot \frac{2x^5}{2x^5} = \frac{3}{2x^2}.$$

17. Explain the example above.

18. Write in lowest terms.

a. $8x^8/6x^9$ b. $7x^7/5x^4$

In some cases, a ratio can be simplified to a monomial. For example,

$$\frac{150x^6}{50x^4} = 3x^2.$$

19. a. Explain this example.

b. Write $3x^2$ as a ratio of monomials in three other ways.

20. Write $12y^3$ as a quotient of two monomials in which

a. one is a fourth-degree monomial;

b. one has a coefficient of 5;

c. one is a monomial of degree 0.

21. Write $1.2 \cdot 10^4$ in three ways as the quotient of two numbers in scientific notation.

22. a. Write x^5 as a ratio in three ways.

b. Find three ratios equivalent to $1/x^5$.

23. **Generalization** Study your answers to problem 22. Compare the situations in (a) and (b). Explain how to simplify a ratio whose numerator and denominator are powers of x.

24. Fill in the exponent, assuming $p > q$.

$$\frac{x^p}{x^q} = x^?$$

25. Write these ratios in lowest terms.

a. $3x^5/6x^4$ b. x^5/y^7

c. y^3/y^7 d. $45a^4/9a^3$

26. Write as a power of 6. $\frac{6^{x-5}}{6^{5-x}}$

SOLVING EQUATIONS

Solve for x.

27. a. $\frac{5^{2x}}{5^x} = 5^7$ b. $\frac{(7^2)^x}{7^4} = 7^6$

28. a. $\frac{(3 \cdot 5)^3}{108 \cdot 5^x} = \frac{1}{20}$ b. $\frac{3^3 \cdot 4^7}{3 \cdot 4^x} = \left(\frac{3}{4}\right)^2$

29.

a. $\frac{3 \cdot 4^{6p}}{9 \cdot 4^x} = \frac{1}{3 \cdot 4^{4p}}$ b. $\frac{15h^x}{12h^a} = \frac{5}{4h^6}$