LESSON 9.7

Halfway Measures

TWO ACCOUNTS

1. **Exploration** Janet and Marne had savings accounts. Marne was earning simple interest, and Janet was earning compound interest. Surprisingly, both accounts grew from \$650.00 to \$805.24 in four years. What was the annual interest rate for each account?

2. How much money was in each account after two years?

3. One account increased by the same *amount* every two years. What was the amount?

4. The other account increased by the same *percent* every two years. What was the percent?

5. **Summary** One account was an example of linear growth, the other was an example of exponential growth. In equal time intervals, one account showed constant differences, while the other showed constant ratios. Explain.

THE MIDPOINT OF EXPONENTIAL GROWTH

Dick and Stan had data about the population of their school. There were 325 students in 1980 and 742 students in 1988. They wanted to estimate the population in 1984.

Dick assumed that the population had grown linearly. This means that for equal time intervals the difference in population would be the same. Algebraically,

$$P_{1984} - P_{1980} = P_{1988} - P_{1984}.$$

6. Use algebra to find P_{1984}.

7. If Dick's assumption was correct, what was the population in 1986?

Stan assumed that the population had grown exponentially. This means that for equal time intervals, the population ratios would be the same. Algebraically,

$$\frac{P_{1984}}{P_{1980}} = \frac{P_{1988}}{P_{1984}}$$

$$\frac{P_{1984}}{325} = \frac{742}{P_{1984}}$$

8. Solve for P_{1984}. (Hint: Multiply both sides by 325 and then by P_{1984}.)

9. If Stan's assumption was correct, what was the population in 1986? Explain your reasoning and show your calculations.

10. Assume Stan's assumption was correct and also that the population grew at the same rate from 1980 to 1992. Make a table showing an estimate of the population at two-year intervals during this time period.

LINEAR OR EXPONENTIAL?

Solve these problems in two ways, assuming

a. that the growth is linear;

b. that the growth is exponential.

c. Discuss which assumption is more reasonable, or whether neither one is credible.

11. A tree was 6 feet high in 1930 and 21 feet high in 1980. How high was it in 1955?

12. A tumor was estimated to weigh about 4 grams in January and 7 grams six months later. If it continued to grow in the same way, how much would it weigh after three more months?

13. **Generalization** A growing population is P_1 at a certain time and P_2 at a later time. Use algebra to find its size halfway between these two times, assuming

a. linear growth;

b. exponential growth.

USING AN EQUATION

A population grew from 1000 to 2197 in three years.

14. Assume linear growth.

a. How much did the population grow each year?

b. Make a table showing the population at the end of one, two, three, and four years.

c. Write an equation expressing the population as a function of the number of years.

d. Use the equation to find out the population after 27 months. (Hint: First figure out how many years that is.)

15. Assume exponential growth.

a. By how much was the population multiplied each year?

b. Make a table showing the population at the end of one, two, three, and four years.

c. Write an equation relating the population to the number of years.

Your equation should be in the form $P = 1000b^x$, with x indicating the number of years.

16. Use the equation and your calculator to find the population after:

a. 27 months; b. 2.5 years;

c. 1 month.

REVIEW/PREVIEW

CALCULATOR PREDICTIONS

17. a. Predict how your calculator will respond if you try to use it to compute $\sqrt{-9}$.

b. Explain your prediction.

c. Check whether you were right.

For each problem, 18-24, two expressions are given.

a. Predict which is greater or whether they are equal.

b. Explain your prediction.

c. Use your calculator to check whether you were right.

18. $\sqrt{2} + \sqrt{8}$ or $\sqrt{18}$

19. $\sqrt{27}$ or $3\sqrt{3}$

20. $2\sqrt{3}$ or $\sqrt{2 \cdot 3}$

21. $\sqrt{3} + \sqrt{3}$ or $\sqrt{6}$

22. $\sqrt{2}/\sqrt{3}$ or $\sqrt{2/3}$

23. $\sqrt{2}\sqrt{3}$ or $\sqrt{2 \cdot 3}$

24. $\sqrt{3} + \sqrt{3} + \sqrt{3}$ or $3\sqrt{3}$

25.

a. Predict how your calculator will respond if you try to use it to compute $49^{.5}$ (49 to the power one-half).

b. Explain your prediction.

c. Use your calculator to check whether you were right.