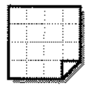
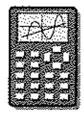
LESSON 13.8

Quadratic Equations: $x^2 + bx + c = 0$

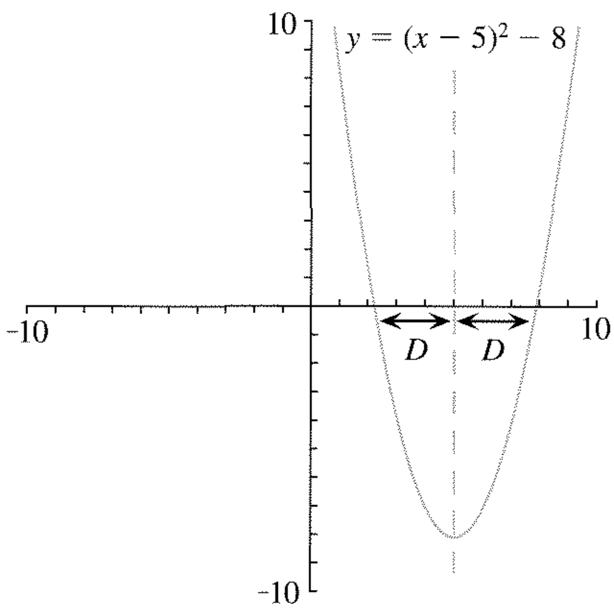
You will need:

graph paper 

graphing calculator (optional) 

FINDING THE x-INTERCEPTS

Earlier in this chapter you learned how to find the vertex after finding the x-intercepts. In this section you will learn how to find the x-intercepts after finding the vertex.



1. **Exploration** As the figure shows, the x-intercepts are equidistant from the axis of symmetry. How can you tell how far they are from it? That distance is indicated by D on the figure. Is it possible to know the value of D by looking at the equation? Try several values for H and V in equations having the form $y = (x - H)^2 + V$. Look for a pattern.

The graph of each of the following quadratic functions is a translation of $y = x^2$. For each function:

a. Make a rough sketch of the graph. Draw and label the axis of symmetry.

b. Find H and V.

c. Find exact values, not approximations, for the x-intercepts. (Set $y = 0$ and use the equal squares method.)

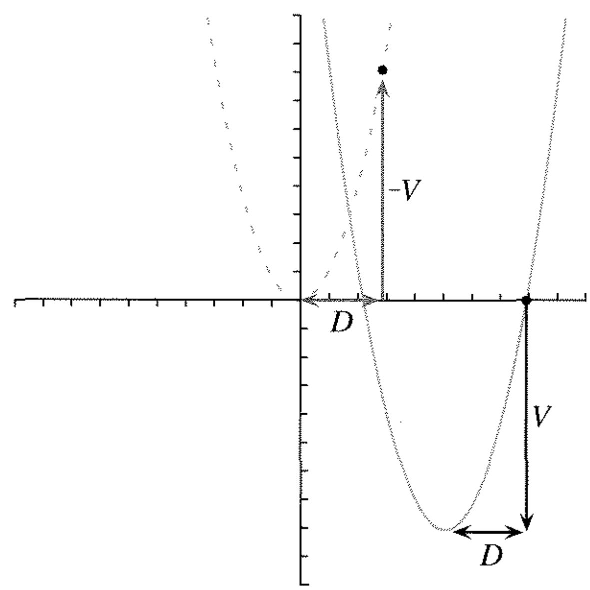
d. Find D, the distance of each x-intercept from the line of symmetry.

2. $y = x^2 - 9$ **3.** $y = (x - 5)^2 - 9$

4. $y = (x - 9)^2 - 5$ **5.** $y = (x + 9)^2 - 5$

6. $y = (x + 9)^2 + 5$ **7.** $y = (x - 9)^2$

8. Use patterns in problems 2-7 to explain how D and V are related.



This figure shows D and V on a parabola that was translated from $y = x^2$. In this example, V was a negative number, and the translation was in a downward direction. The arrows representing D and V are also shown on the original parabola. (On $y = x^2$, the direction of the arrow for V was reversed. What is shown is actually the opposite of V. This is indicated by the label $-V$. Since V is negative, $-V$ is positive.)

9. Use the figure to explain why $-V = D^2$, and therefore $D = \sqrt{-V}$.

10. **Summary** Explain why the x-intercepts, when they exist, are equal to $H - \sqrt{-V}$ and $H + \sqrt{-V}$.

SOLVING QUADRATIC EQUATIONS

One way to solve the equation $x^2 + bx + c = 0$ is to find the x-intercepts of $y = x^2 + bx + c$. You can use a graphing calculator to find an approximate answer that way. For a precise answer, you can use what you learned in the previous section about how to find the x-intercepts from the vertex.

Example: Solve $x^2 + 4x + 1 = 0$.

The solutions to the equation are the x-intercepts of $y = x^2 + 4x + 1$. We have shown that they are equal to $H - \sqrt{-V}$ and $H + \sqrt{-V}$. So all we have to do is find the values of H and V. There are two ways to do that, outlined as follows:

First method: Find H and V by rewriting the equation $y = x^2 + 4x + 1$ into vertex form. This can be done by completing the square.

$$y = x^2 + 4x + 1 = \text{(a perfect square)} - ?$$
$$y = x^2 + 4x + 1 = (x^2 + 4x + \ldots) - ?$$
$$y = x^2 + 4x + 1 = (x^2 + 4x + 4) - 3$$

11. a. Explain the algebraic steps in the three preceding equations.
    b. Write $y = x^2 + 4x + 1$ in vertex form.
    c. Give the coordinates of the vertex.

Second method: Find H and V by first remembering that $H = -(b/2)$. In this case, $b = 4$, so $H = -(4/2) = -2$. H is the x-coordinate of the vertex. Since the vertex is on the parabola, we can find its y-coordinate, V, by substituting -2 into the equation.

12. a. Find V. Check that it is the same value you found in problem 11.
    b. Now that you have H and V, solve the equation.

13. What are the advantages and the disadvantages of each method? Explain.

For each equation, find H and V for the corresponding function. Then solve the equations. There may be zero, one, or two solutions.

14. $y = x^2 + 6x - 9$
15. $y = x^2 - 6x + 9$
16. $y = x^2 - 6x - 9$
17. $y = x^2 + 6x + 12$

18. How does the value of V for the corresponding function affect the number of solutions? Explain.

QUADRATIC EQUATIONS CHECKPOINT

As of now you know five methods to solve quadratic equations in the form $x^2 + bx + c = 0$. They are listed below.

I. *On Graphing Calculators:* Approximate solutions can be found by looking for the x-intercepts of $y = x^2 + bx + c$.

II. *Factoring* and the zero product property can sometimes be used.

III. *Equal Squares:* First complete the square, then use the equal squares method.

IV. *Using Vertex Form:* Complete the square to get into vertex form, then use the fact that the solutions are equal to $H - \sqrt{-V}$ and $H + \sqrt{-V}$.

V. *Using the Vertex:* Remember that for the function $y = x^2 + bx + c$, $H = -b/2$. Substitute into the equation to find V. Then use the fact that the solutions are $H - \sqrt{-V}$ and $H + \sqrt{-V}$.

Caution: In the next chapter you will learn another way to solve quadratic equations in the more general form $ax^2 + bx + c = 0$. Meanwhile you can solve them by dividing every term by a.

Example: Find an exact solution for:

$$x^2 - 6x + 2 = 0.$$

This does not seem to factor easily, which rules out Method II, and an exact solution is required, which rules out Method I. Luckily, Methods III-V always work on problems of this type. Using Method III:

$$x^2 - 6x + 2 = 0$$
$$(x^2 - 6x + 9) - 7 = 0$$
$$(x - 3)^2 - 7 = 0$$
$$(x - 3)^2 = 7$$

So $x - 3 = \sqrt{7}$ or $x - 3 = -\sqrt{7}$, and the solutions are $3 + \sqrt{7}$ and $3 - \sqrt{7}$.

19. Solve the same equation with Method IV or V. Check that you get the same answer.

Solve these equations. Use each of Methods II-V at least once. Give exact answers. The equations may have zero, one, or two solutions.

20. $x^2 - 4x + 2 = 0$

21. $x^2 + 8x - 20 = 0$

22. $x^2 - 14x + 49 = 0$

23. $x^2 - 16x + 17 = 0$

24. $x^2 + 9x = 0$

25. $x^2 + 9 = 0$