Chapter 9 More Equation Solving

This chapter extends what was learned in chapters 7 and 8 by using those techniques to solve more challenging equations.

New Words and Concepts

The Lab Gear techniques for solving **simultaneous equations** are a natural extension of the work done in solving linear equations. Substitution is really the only convenient Lab Gear method, and you will need to teach the other methods (graphic solutions and linear combinations) some other way. Nevertheless, the exercises in this section are very valuable, as they establish some basic concepts about simultaneous equations.

As for quadratics, after reviewing factoring, this chapter builds on past "Make a Square" activities, to lead the students to learn about **completing the square**. This lays the foundation for a solid understanding of the derivation of the quadratic formula, the traditional climax of first year algebra.

Teaching Tips

The final lessons are all quite challenging, and are a way to bring together everything that was learned throughout the Algebra Lab program. This would be a good time to work on any unresolved "Always, Sometimes, or Never?" questions that may be left over from previous chapters.

Lesson Notes

- **Lesson 1,** Simultaneous Equations, page 114: It requires two workmats to work each of these problems, so make sure you have duplicated enough copies.
- **Lesson 2,** Zero Products, page 118: A chance to review material from Chapter 8.
- **Lesson 3,** Equal Squares, page 118: Another approach in the first example is to subtract 25 from both sides, and then factor the left side. That method, based on the difference of squares identity can be used for all "equal squares" problems.
- **Lesson 4,** Completing the Square, page 120: Make sure the students remember that "equal squares" problems usually have two solutions.

Exploration 1 Solving for y

Set up these problems with the Lab Gear, and rearrange them so that y is by itself on the left. Write equations to show all the steps. In some cases, you will need to finish the problem without the blocks.

1. $y - 4x = 6$
2. $2y + 4x - 10 = 0$
3. $-6x + y = 4$
4. $3y - 6x = 9$
5. $6x - 3y = 12$
6. $2y + x = 8$
7. $x - y = 1$
8. $6x - 5y = 0$

Exploration 2 Solving for x and y

1. a. Set up this equation with the blocks: $2x + y = 9$.
   Using trial and error, find some values of x and y that make it true.
   b. Suppose you are told that x is 3 more than y. Were any of your answers to problem 1 correct? If not, can you solve the equation correctly? (Hint: Set up the equation with only one variable.)

For each of these problems, set up the equation with the blocks, then use the extra fact to solve the equation. Write the value of x and y.

2.	$4x - 7 = y + 3$	y is 2 more than x
3.	$2y + x = 5$	x is 6 less than y
4.	$3y + 2x = 12$	y is twice x
5.	$2y + x = 4$	x and y add up to 6
6.	$4x - 2y = 5$	y is one-third of x
7.	$2x = 6 + y$	x divided by y equals 5
8.	$x^2 - 2x + y = 0$	y is the square of x

Try solving these equations without the Lab Gear.

9.	$y + \frac{3}{x} = 2$	x is the reciprocal of y
10.	$\frac{1}{x-1} = \frac{2}{y}$	y is 2 more than 3 times x
11.	$6 - \frac{3}{x-2} = \frac{3}{y}$	y is 2 less than x
12.	$\frac{x-1}{2} = \frac{1}{2y}$	x is twice y (Hint: there is more than one possible answer.)

Exploration 3 More Solving With Two Variables

Use the Lab Gear to set up the first equation. Then, use the extra fact to substitute other blocks for the *x*-blocks or the *y*-blocks.

1. $2x - y = 2$	$y = 3x$
2. $4x + y = 10$	$y = 6x - 20$
3. $x - 4y = 23$	$x = -5y - 4$
4. $3y + 2x = 7$	$3y = 4x - 5$
5. $5y - 4x = -9$	$5y = 3x - 7$

For the next problems, you may need to rewrite the second equation for it to be helpful.

6. $5x + 3y = -15$	$y = 2x + 6$
7. $5x - 3y = -29$	$x = 2 - 2y$
8. $2x + 3y = 9$	$4x = 6 - 2y$
9. $4x - y = 5$	$3y = 6x + 3$
10. $x - 3y = -18$	$2x + 3y = 9$
11. $6x - 2y = -16$	$4x + y = 1$
12. $3x + 5y = 17$	$2x + 3y = 11$

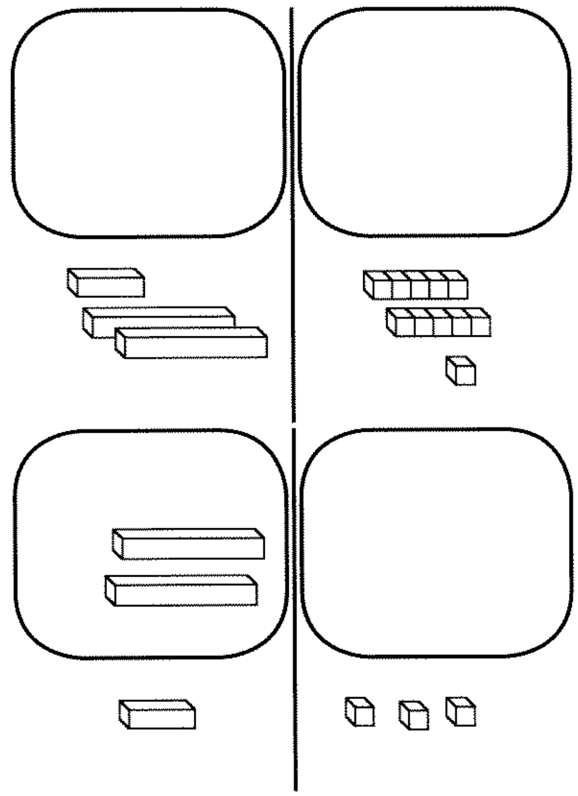
Lesson 1

Simultaneous Equations

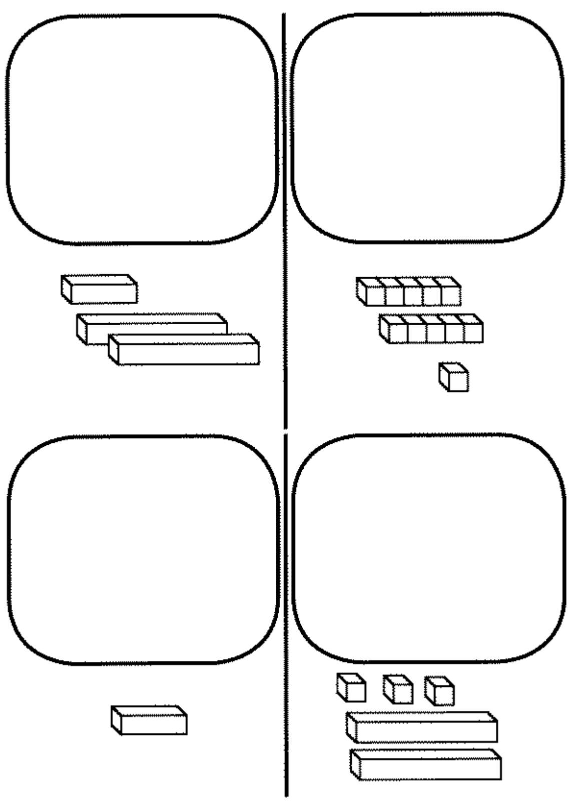
1. Look at this equation: $x + 2y = 11$. Try solving for x.
   Use the Lab Gear, pencil and paper, or mental arithmetic to answer these questions.
   a. If $y = 0$, then $x =$ ______?
   b. If $y = 1$, then $x =$ ______?
   c. If $y = -2$, then $x =$ ______?
   d. If $y = 0.5$, then $x =$ ______?
   e. Write the number of solutions you think there are for this equation.
2. Find four sets of values for x and y that are solutions to the equation $x - 2y = 3$. Try each of them to see whether it is also a solution to the equation in problem 1.

These figures show how to use the Lab Gear to find a set of values for *x* and *y* that satisfy two equations *at the same time.*

- You will need two workmats. Show one equation on each workmat.



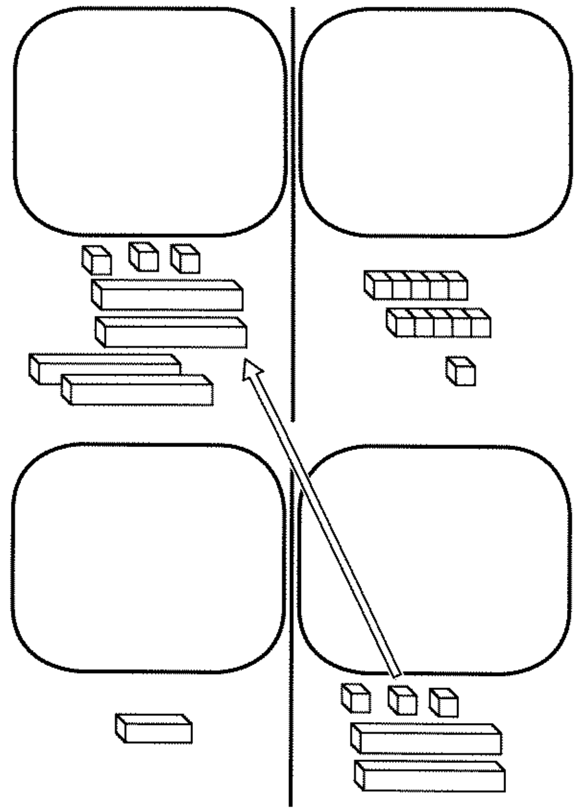
3. Write the two equations.

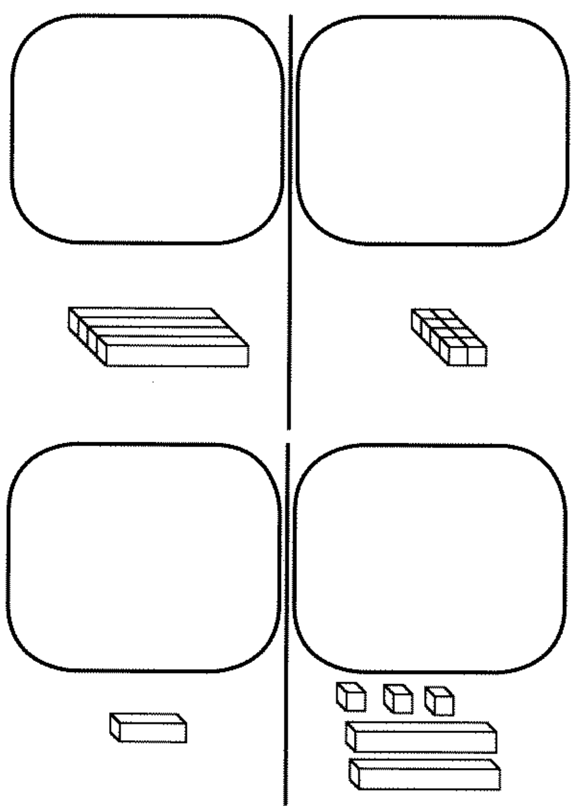


- Next, on one of the workmats, solve for one of the variables in terms of the other. In this case, the bottom workmat is used, and *x* is evaluated in terms of *y.*

4. Write the value of *x* in terms of *y*, for the bottom workmat.

- Next, x is replaced by $2y + 3$ on the other workmat. (Notice that additional blocks are used. Do not simply slide blocks from one workmat to the other—you will soon see why.)



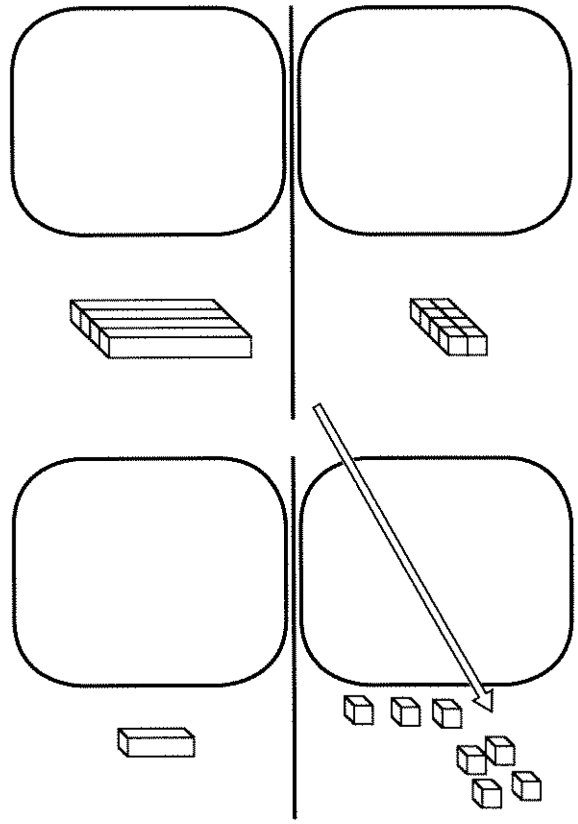


- Now solve for y on the upper workmat. This is straight-forward, as there are no x-blocks left to get in the way.

5. Write the value of y.

- Finally, substitute the value you found for *y* into the lower workmat, and solve the equation there. Again, this is easy to solve, since there are no *y*-blocks left.

6. Write the value of *x*.
7. Check that the values you found for *x* and *y* are indeed solutions to both original equations.



Solve these systems of **simultaneous equations** with the Lab Gear. Some systems will have a single common solution, as in the example. One will have no common solution. One will be such that whatever numbers work for one equation, also work for the other.

8. $3x + 8y = 20$
   $3x + y = 13$
9. $2x + y = -1$
   $x - 3y = -18$
10. $6x - 2y = 12$
    $y = 3x - 6$
11. $x - y = 1$
    $x + 3y = -11$
12. $4x - y = 2$
    $y = 4x + 1$
13. $y = 7x + 10$
    $y = 4 + x$

☑ Self-check

8. $y = 1$, $x = 4$
12. No solution

Zero Products

Solve these equations, using factoring and the Zero Product Principle. One cannot be factored.

1. $9x^2 + 12x + 4 = 0$
2. $2x^2 + 9x + 10 = 0$
3. $4x^2 - 9 = 0$
4. $6x^2 - 4x - 2 = 0$
5. $4x^2 + 10x = 0$
6. $2x^2 + 10x + 5 = 0$
7. $x^2 - 2x + 1 = 0$
8. $9x^2 - 21x + 10 = 0$

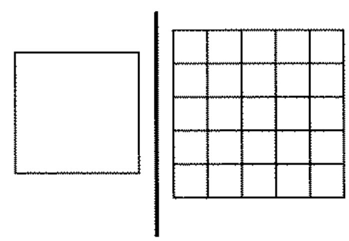
☑ Self-check

8. $x = \frac{2}{3}$ or $x = \frac{5}{3}$

Lesson 3

Equal Squares

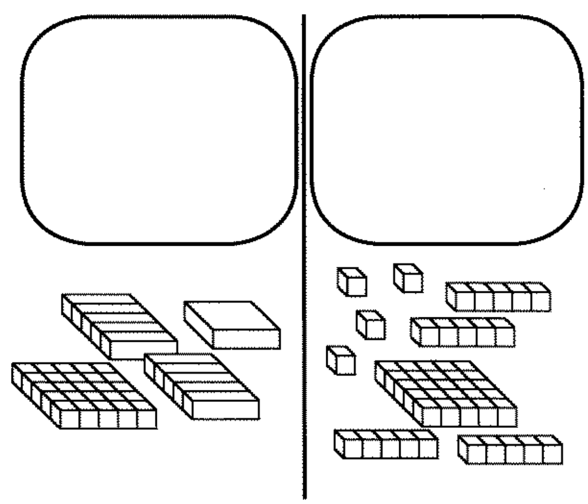
Consider the equation $x^2 = 25$. Put out your blocks like this to illustrate it.



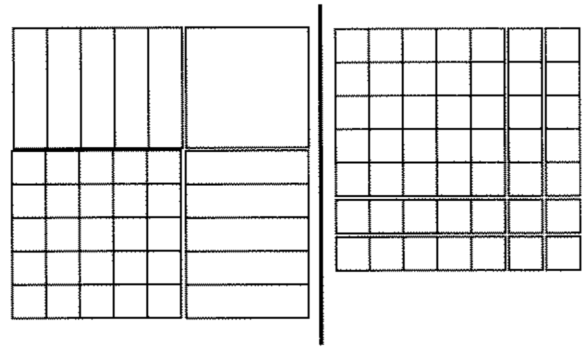
One way to solve this equation is to remember that if the squares are equal, *their sides must be equal.* (This is true even though they don't *look* equal. Remember x can have any value.)

1. Solve the equation. If you only found one solution, think some more, because there are two.
2. Explain why there are two solutions.

3. Write the equation shown by this figure.



By rearranging the blocks, you can see that this is an *equal squares* problem, so it can be solved the same way.



As you can see in the figure, $(x + 5)^2 = 7^2$. It follows that $x + 5 = 7$, or $x + 5 = -7$.

4. Solve the equation. There are two solutions. Check them both in the original equation.

Solve the following equations using the equal squares method. Most, but not all, have two solutions (in one case the solutions are not whole numbers).

5. $x^2 = 16$
6. $x^2 + 2x + 1 = 0$
7. $4x^2 = 36$
8. $4x^2 - 4x + 1 = -9$
9. $4x^2 - 4x + 1 = 9$
10. $y^2 - 10y + 25 = 5$
11. Explain why some problems had one, or no solution.

To solve this problem, remember that if quantities are equal, *their opposites must be equal.*

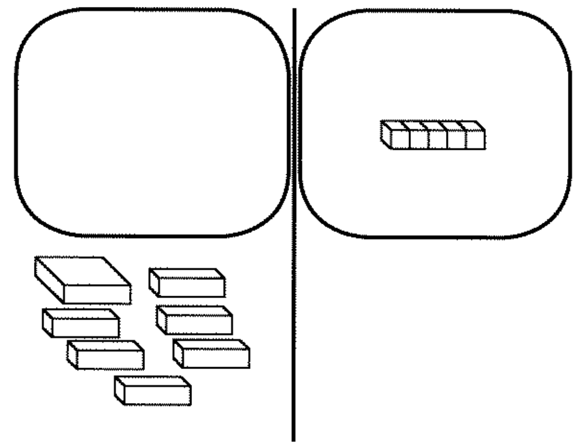
12. $-x^2 - 6x - 9 = -25$

☑ Self-check

9. $x = 2$, or $x = -1$

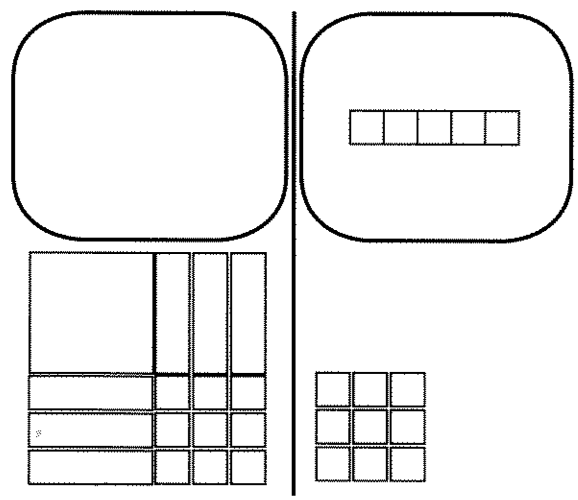
Completing the Square

1. Write the equation shown by this figure. Put out your blocks like this to illustrate it.



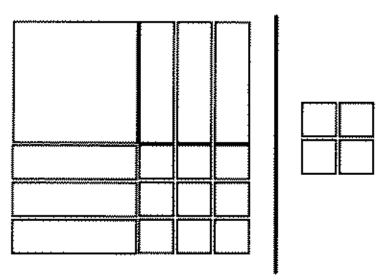
By adding the same quantity to both sides, it is possible to change this equation into an *equal squares* problem. This is called **completing the square**.

2. Write the quantity that was added to complete the square on the left.



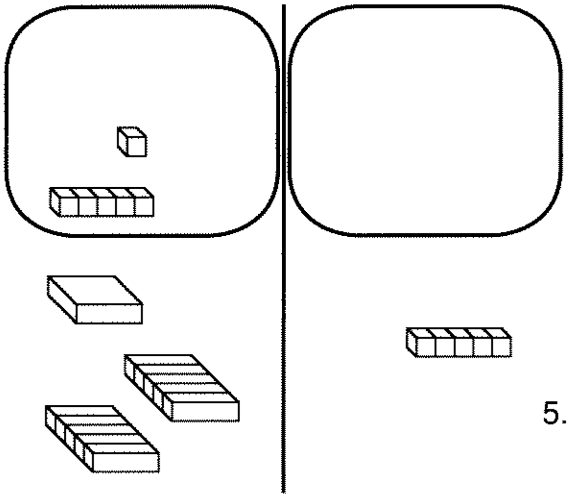
The figure shows what is left after cancelling on the right.

3. Solve the equation. There will be two solutions.

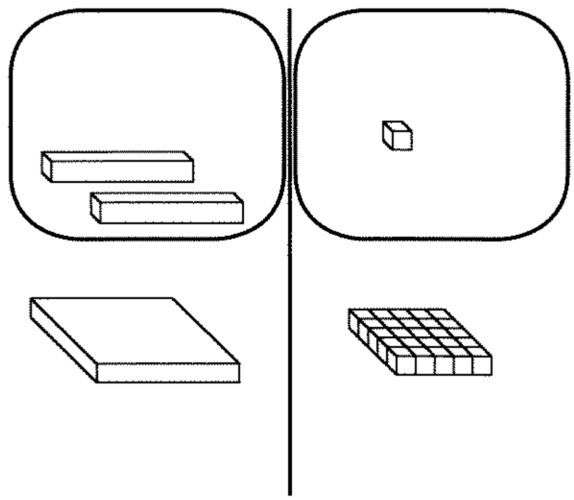


For the following equations, you will need to rearrange the blocks, and add or subtract the same amount on both sides, in order to get equal squares.

4.



5.



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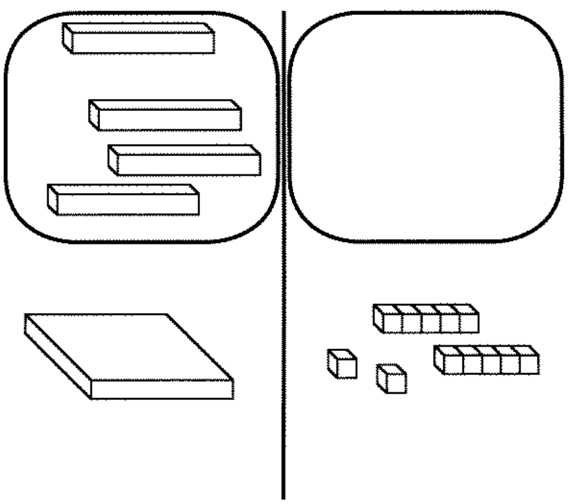


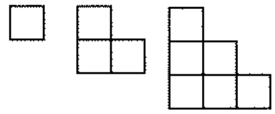
7. $y^2 - 6y = -9$
8. $y^2 = 8y - 7$
9. $-y^2 = 8y + 7$
10. $x^2 + 6x = -9$

☑ Self-check

9. $y = -1$, or $y = -7$

Exploration 4 Area and Perimeter

Look at this sequence. Think about how it continues, following the pattern. Write the perimeters of the figures given, then the perimeter of the fourth one, the tenth one, and the hundredth one. Explain how you got your answers.



Exploration 5 Volume and Surface Area

Look at this sequence. Think about how it continues, following the pattern. Write the volume and surface area of each figure given, then the volume and surface area of the fourth one, the tenth one, and the hundredth one. Explain how you got your answers.