Negative Exponents — Teacher Notes

This is the 8th grade version of the part of the Algebra 1 packet on **Powers**. It is based on *Algebra: Themes, Tools, Concepts,* lessons 8.11-8.12. It follows the 8th grade **Powers** packet, and the **Scientific Notation** packet. All these packets are available on www.MathEducationPage.org/middle-school/

Lesson 1: Negative Exponents

The meaning of negative exponents is derived from patterns, and from the product of powers law.

Lesson 2: Reciprocals and Opposites

Negative bases and parentheses are the source of many mistakes in algebra. A discussion of #3 and #4 might help throw some light on that subject.

Lesson 3: Ratio of Powers

Here the ratio of powers is reviewed, and applied in cases where simplifying ratios yields negative exponents.

Lesson 4: More on Exponential Growth

This lesson is based on Lessons 1 and 2 from the 8th grade **Powers** packet. Here, we apply negative exponents to make estimates of past values.

Lesson 5: Negative Exponents of 10

This lesson applies the idea of negative exponents to powers of 10, and thus to scientific notation. While the unit tries to give a solid conceptual foundation for this, it is important to give the students some practice.

Doing #9 is preferable to applying the ideas to teacher-supplied examples. See the teacher notes for a similar assignment in the **Scientific Notation** packet.

Lesson 1: Negative Exponents

In previous lessons, we have considered only positive whole number exponents. Does a negative exponent have any meaning? To answer this, consider these patterns:

$3^4 = 81$	$(1/3)^4 = 1/81$
$3^3 = 27$	$(1/3)^3 = 1/27$
$3^2 = 9$	$(1/3)^2 = 1/9$
$3^1 = 3$	$(1/3)^1 = 1/3$
$3^0 = ?$	$(1/3)^0 = ?$
$3^{-1} = ?$	$(1/3)^{-1} = ?$

1. a. Look at the powers of 3. How is each expression related to the expression above it? Explain.

   b. Following this pattern, what should the value of 3^{-1} be?

   c. Now look for a pattern in the powers of 1/3. As the exponent increases, does the value of the expression increase or decrease?

   d. Following this pattern, what should the value of $(1/3)^{-1}$ be?

   e. Compare the values of 3^{-1}, 3^1, $(1/3)^1$ and $(1/3)^{-1}$. How are they related?

   f. Use the pattern you found to extend the table down to 3^{-4} and $(1/3)^{-4}$.

Another way to figure out the meaning of negative exponents is to use the *product of powers law*:

$$x^p \cdot x^q = x^{p+q}$$

For example, to figure out the meaning of 3^{-1}, note that:

$$3^{-1} \cdot 3^2 = 3^1$$
$$3^{-1} \cdot 9 = 3$$

So 3^{-1} must equal 1/3.

2. Confirm the value of 3^{-1} by applying the product of powers law to $3^1 \cdot 3^{-1}$.

3. Use the same logic to find the value of

   a. 3^{-2}

   b. 3^{-x}

4. Are the answers you found in problem 3 consistent with the pattern you found in Problem 1? Explain.

Lesson 2: Reciprocals and Opposites

Reciprocals

1. Many people think that 5^{-2} equals a negative number, such as -25.

   a. Write a convincing argument using the product of powers law to explain why this is not true.

   b. Show how to find the value of 5^{-2} using a pattern like the one in problem 1.

The product of reciprocals is always 1. For example, $1/3 \cdot 3 = 1$.

2. a. What is the reciprocal of 9^3 ?

   b. What is the reciprocal of 9^{-8} ?

   c. What is the reciprocal of a^b ?

Opposites

The expression $(-5)^3$ has a negative base. This expression means *raise -5 to the third power*.

The expression -5^3 has a positive base. This expression means raise *5 to the third power and take the opposite of the result*.

3. Which of these expressions have negative values? Show the calculations or explain the reasoning leading to your conclusions.

-5^3 $(-5)^3$ -5^2 $(-7)^{15}$ $(-7)^{14}$

-5^{-3} $(-5)^{-3}$ -5^{-2} $(-7)^{-15}$ $(-7)^{-14}$

4. a. Is $(-5)^n$ always, sometimes, or never the opposite of 5^n? Explain, using examples.

b. Is -5^n always, sometimes, or never the opposite of 5^n? Explain, using examples.

Lesson 3: Ratio of Powers

Negative exponents often arise when simplifying ratios of monomials.

This law of exponents is sometimes called the *ratio of powers* law:

$$\frac{x^a}{x^b} = x^{a-b}$$, as long as x is not 0.

However, notice that it works only when the bases are the same.

Examples

$$\frac{x^6}{x^7} = x^{6-7} = x^{-1} \text{ or } \frac{1}{x^1}$$

$$\frac{x^{3a}}{x^{5a}} = x^{3a-5a} = x^{-2a} \text{ or } \frac{1}{x^{2a}}$$

1. Simplify:

   a. $4x^6 / 5x^7$

   b. $2x^8y^3 / 2xy$

   c. y^3 / y^7

   d. $45a / 9a^5$

2. Simplify:

   a. $\dfrac{400a^5}{25a^2}$

   b. $\dfrac{400x^3}{200x^8}$

   c. $\dfrac{3m^6}{9m^3}$

   d. $\dfrac{9R^a}{3R^a}$

Lesson 4: More on Exponential Growth

A bacterial culture doubles every hour. At this moment it weighs 16 grams.

1. What will it weigh
   a. in one hour?

   b. in 2 hours?

   c. in 9 hours?

2. Explain how to calculate what the bacterial culture will weigh in *x* hours. **Hint**: A good way to explain this is to use powers of 2. Check that your idea works for the questions in #1.

3. What did the bacterial culture weigh
   a. 1 hour ago?

   b. 2 hours ago?

   c. 4 hours ago?

4. Explain how to calculate what the bacterial culture weighed *x* hours ago.

5. Explain how to answer question #4 by using multiplication. **Hint:** Use powers of ½.

6. Explain how to answer question #4 by using multiplication and powers of 2. **Hint:** Use negative exponents.

Lesson 5: Negative Powers of 10

1. Fill out this table:

$10^4 = 10{,}000$	$(1/10)^4 = 1/10{,}000 = 0.0001$
$10^3 =$	$(1/10)^3 =$
$10^2 =$	$(1/10)^2 =$
$10^1 =$	$(1/10)^1 =$
$10^0 =$	$(1/10)^0 =$
$10^{-1} =$	$(1/10)^{-1} =$

2. Explain how to find 10^n without a calculator
   a. if n is positive
   b. if n is 0
   c. if n is negative

3. Using a power of ten, write the reciprocal of each number.
   a. 10^2
   b. 10^{-4}
   c. 0.001
   d. 100

4. Write 4321000 in scientific notation. (Remember that scientific notation requires multiplying a number between 1 and 10 by a power of 10.)

5. Write 0.065 in scientific notation. **Hint:** this requires negative exponents!

6. Write these numbers without exponents:
   a. $7.8 \cdot 10^6$
   b. $7.8 \cdot 10^{-6}$

7. Write these numbers in scientific notation:
   a. 9012
   b. 0.0123

8. **Summary:**
   a. Explain how to convert very large numbers into scientific notation.
   b. Explain how to convert very small numbers into scientific notation.
   c. Explain how to convert a number from scientific notation to a simple decimal number.

9. **Research.** Find four very small numbers that measure some real quantity. They should all be smaller than 1/1000. The Web, encyclopedias, almanacs, and science books are good sources of such numbers.

   a. Tell what each number measures.

   b. Write each number in scientific notation.