Percent — Teacher Notes

Philosophy

The basic philosophy of these lessons is to *teach for understanding*. Thus:

- The lessons start by describing a situation *without invoking new vocabulary or notation*. This could be a simplified real world context, or just a number pattern. The main point is not so much that they will necessarily discover new concepts, but that they will be better prepared to hear a teacher explanation. If some of them discover the concepts, all the better.
- The particular topics of percent and powers both require *extensive use of the calculator*. This is consistent with the Common Core emphasis on smart use of tools. See the note below on possible exceptions to this.
- Once the concept and any related techniques are introduced, students apply them to other problems.
- Some of the problems involve making a table of change over time. Other problems involve trial and error (guess and check, if you prefer.) These kinds of problems are a form of *practice* that is not boring, or at any rate, less boring than random exercises.
- There is a fair amount of *repetition*, because the ideas are difficult, but hopefully not exact repetition.
- Because some of the work is challenging, allow the students to help each other, and be ready to help them as well, sometimes individually, sometimes through a class discussion.

Once new rules have been learned, post them prominently in the classroom. This way the focus will be on understanding, not memorization. You may at some later date decide to ask for memorization and take them down, but starting with memorization is a recipe for disaster. The kids who can do it feel they can turn off their brain and need not understand what they're doing. This sort of mastery is fragile and does not last. And the kids who have trouble memorizing things they don't understand are frozen out of the lesson altogether. Neither group benefits if you give up on teaching for understanding.

No-Calculator Activities

While the calculator is assumed throughout these lessons, you can and should do some no-calculator activities along the way. For example you can ask the students to figure out mentally basic percent and power questions, like:

- What is 10% of 35? (you can do a bunch of the easiest ones before you get fancier — 10%, 50%, 25%)
- What is 20% or 35? (note how this one builds on the previous one)
- etc.

The purpose of the mental math is two-fold: on the one hand, it shows that one need not run to the calculator every time; on the other hand, it helps kids focus on the meaning of percent in a way that just working on paper and with the calculator cannot do.

CCSS-M

This series of lessons on percent is designed to address the following Grade 7 Common Core standards:

7. RP 3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*.

7. NS 3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Because these materials are designed to foster teaching for understanding they naturally hit on other CCSS standards for Grade 7 (for example, other aspects of 7.NS.) They also revisit Grade 6 (6.EE 9) and prepare students for Grade 8 (8.EE and 8.F) and Algebra 1 (A-SSE and F-LE.)

Lesson 1: An Algebra Tutor's Salary

This is an introduction to percent increase, as compared to linear increase.

It is probably a two-day lesson.

Because the point is to compare the two salary agreements, make sure to start by introducing both up front, writing notes on the board. You might ask students to rephrase each tutor's salary agreement, and to guess which is the better deal. It is important that students understand the situation under discussion before they dive into the worksheet.

While the students are working, stop them periodically to discuss the most important questions as a whole class. In this lesson, it would be great to hear many ideas about #1, #3, #5, #6, #8, #10.

Lesson 2: Calculating with Percents

The big idea here is far from obvious: how to figure out total cost including a mark-up, or a sales tax, or both, by using multiplication.

After the students have started with the worksheet, many will probably be confused. That is not a bad thing — it is absolutely normal, and it sets up the needed situation for a conversation. You can use Mr. Carr's strategy to explain how it's done. If your students, or some of them, are familiar with the distributive rule, you can mention it, and use this lesson as an opportunity to review this idea.

For #3 and #5, encourage trial and error (guess and check.) Do not expect the students to know how to do it by algebra! Trial and error here is great practice, and it is motivated by trying to get an answer.

Lesson 3: After Halloween

This is an introduction to percent decrease. Again, the key idea is that one can do these calculations with just multiplication. However, do not reveal this at the start. Instead, save that strategy for when most students have completed #1 and #2. At that point, they will be much readier to understand the shortcut, and they can use it for #5.

Lesson 4: Discounter Introduces Reductions

(Non-mathematical aside: the title of this lesson consists of three words that are all anagrams of each other.)

The start of the lesson is a review of how to calculate percent decrease by multiplication. (That idea was introduced in Lesson 3.) It leads to the challenging problems in #3-6. Let students struggle with those for a bit. Have them explain their approaches. If no one can figure it out, of course, you can give hints. Note that just saying G.D. was cutting the price in half is not enough: students should see the connection with 20%+30%.

#7 does not have one right answer. If the additional 20% off is taken off the original price, G.D. was right. If it's taken off the discounted price, Cal was right. 20% of the original price is of course a bigger discount!

Lesson 1 (two days): An Algebra Tutor's Salary

You will need: calculator, graph paper

Bea did so well in algebra that she got a job as an algebra tutor. Her starting salary, as she had no experience, was \$10 per week. As Bea got more experience, her salary increased. She got a raise of \$1 per week.

Abe also got a job as an algebra tutor. He heard that Bea was getting a weekly raise of \$1. Since \$1 is 10% of \$10, Abe asked for a weekly raise of 10%. The first week Bea and Abe both got the same raise.

Here is a table of Bea's salary in the first weeks.

Weeks	Salary	Amount Increase	Calculating Percent Increase using a calculator	Percent Increase
0	\$10			
1	\$11	\$1	1/10 = 0.1	10
2	\$12	\$1	1/11 = 0.0909	9.09
3	\$13	\$1		8.33
4				
5				
6				
7				
8				
9				
10				

Her first increase was 10%, because it was a \$1 increase on a \$10 salary, and
1/10 = 10/100 = 10%

Her second increase was 9%, because it was a \$1 increase on an \$11 salary, and
1/11 = 0.0909… = 9.09% (approximately.)

1. Finish this sentence with words and calculations:
   Her third increase was 8.33%, because…

2. Complete the table for the first ten weeks that Bea worked.

3. Explain why the number in the Percent Increase column decreases each week.

4. Compare Bea's original salary with her salary for the tenth week.
   a. What was the total amount of increase in her salary?

   b. What percent of her original salary is this total increase? (This is the total *percent increase*.)

   c. What percent of her original salary is her salary in the tenth week? (Your answer should be a number greater than 100. Why?)

5. To get the next week's salary for Bea, do you add the same number or multiply by the same number? What is the number?

Lesson 1 (continued): Percent Increase

Here is a table of Abe's salary in the first few weeks:

Weeks	Salary	Percent Increase	Calculating Amount Increase using a calculator	Amount Increase
0	$10.00			
1	$11.00	10%	$0.10 \cdot 10$	$1
2	$12.10	10%	$0.10 \cdot 11$	$1.10
3	$13.31	10%	$0.10 \cdot 12.1$	$1.21
4				
5				
6				
7				
8				
9				
10				

Abe's second increase was 10% of $11, which is $0.10 \cdot 11 =$ $1.10

His third increase was 10% of $12.10, which is $0.10 \cdot 12.1 =$ $1.21

6. Finish this sentence with words and calculations:
   His fourth increase was 10% of…

7. Complete the table for the first ten weeks that Abe worked.

8. Explain why the numbers in the Amount Increase column increase each week.

9. Compare Abe's original salary with his salary for the tenth week.
   a. What was the total amount of increase?

   b. What is the total *percent increase*?

   c. What percent of his original salary is his salary in the tenth week?

10. **Challenge:** To get the next week's salary for Abe, do you add the same number or multiply by the same number? What is the number? Test your ideas on a calculator.

11. On the same pair of axes, make graphs of Abe's and Bea's weekly salaries as a function of weeks of experience. Compare the graphs. Which is straight? Which is curved?

12. Do either of the graphs of Abe's and Bea's weekly salaries as a function of weeks of experience represent a proportional relationship? Show how you know?

Lesson 2: Calculating With Percents

You will need: calculator

Sales Tax

Sales tax is usually a percent of the cost. In this lesson, you will learn how to find the total cost of things, including sales tax.

Let's say a state has 5% sales tax. If something costs $9.00, you pay the $9 plus 5% of $9. In other words: $9 + 0.05 \cdot 9 = \$9.45$

1. Jenn is thinking of buying a $7 calculator. How much would she have to pay?

Instead of first finding the tax, and then adding it to the cost, Jenn wonders if there's a way to do just one calculation. She asks Mr. Carr, her teacher for help figuring out what she would owe for a fancier calculator, that costs $11.

Mr. Carr points out that if she was buying an $11 calculator, plus 3 more, the cost (not counting tax) would be

$1 \cdot 11 + 3 \cdot 11 = 4 \cdot 11$

So it's just one multiplication.

The same works for sales tax:

$1 \cdot 11 + 0.05 \cdot 11 = 1.05 \cdot 11$

In other words, he explained that the cost including tax is 105% of the original cost. Just one multiplication by 1.05!

2. How much would Mr. Carr have to pay, including tax, to buy
   a. one $11 calculator?
   b. fifteen $11 calculators?
   c. twenty $7 calculators?

3. **Challenge:** If a teacher has $100 to spend on calculators, how many $7 calculators can she get? How many $11 calculators? Don't forget the sales tax!

Mark-up

An electronics store buys calculators from a manufacturer in another state. Each calculator costs them $5. In order to pay for their expenses and to make a profit, they mark up the calculators by a certain percent.

4 If the markup is 50%:
   a. What would the calculator cost, not counting tax?

   b. What would it cost, including tax?

5. **Challenge:** They want to sell the calculator to customers for less than $10, including the sales tax. What is the biggest markup they can use?

Lesson 3: After Halloween

Maria had 40 pieces of candy. She didn't want to eat them all at once. So she made the following rule: each day, she would eat half of what she has left. If she has an odd number left, she will round up and eat a little more than half.

She wonders how long the candy will last, so before starting to eat she made a table like this one:

Day	Candy
0	40
1	20
2	10
3	

1. How many days will the candy last?

Maria decided she wanted the candy to last longer, so she changed her plan. She will eat 20% of the candy she has left each day. If that means she needs to eat a fractional piece of candy, she will round up and eat a little more than 20%.

Again, she wonders how long the candy will last, so before starting to eat she made a table:

Day	Candy
0	40
1	32
2	25
3	

2. Complete the table to show how long the candy will last with this plan.

3. On a certain day, if she eats 20% of the candy, what percent is left?

4. There is a way to calculate the next day's amount of candy by multiplication only, without subtraction. What is it? **Hint**: look at your answer to question 3.

5. Working with your neighbors, find out how many days the candy will last if she changes her rule to eating:
   a. 10%
   b. 30%
   c. 40%
   d. 60%

Lesson 4: Discounter Introduces Reductions

You will need: calculator

Calculators were on sale, 15% off. So the amount that you pay is 15% less than the presale cost.

Example: if a calculator normally cost \$7, you would pay $7 \cdot 0.15 = \$1.05$ less. So the amount that you would pay would be $7 - 1.05 = \$5.95$

1. If the sale is 15% off, you still have to pay 85% of the price.
   a. What is 85% of \$7.00?

   b. What is 85% of \$11.00?

2. Julio wants to buy a calculator that would normally cost \$11. How much would it cost during the 15% off sale? Find the discounted price two ways:
   a. by using multiplication and subtraction, as in the example above.

   b. by using only multiplication.

   c. Did you get the same answer both ways?

Super Sale

Sherman's Department Store ran the following ad in the newspaper.

THREE-HOUR EARLY-BIRD SPECIAL!

This week, all merchandise has been discounted 30% for our year-end clearance sale.

For three hours only, from 9AM to 12 noon on Saturday, get amazing additional savings! We will take an **additional** 20% off the sales price at the cash register.

G.D. and Cal were working during the three-hour sale. At the end of the sale, they compared receipts and discovered that they had sold some of the same items, but they had charged customers different prices for them. They made the following table:

Original price	Cal charged	G.D. charged
$140	$78.40	$70.00
$50	$28.00	$25.00
$19	$10.64	$9.50
$5	$2.80	$2.50
$180	$100.80	$90.00

Before answering the next questions, make sure you have read the "early-bird special" information carefully. Both Cal and G.D. read it, but they understood it differently!

3. How do you think Cal came up with a sale price of $78.40 on an item that originally cost $140?

4. How do you think G.D. came up with a sale price of $70.00 on an item that originally cost $140?

5. Check some of Cal's other charges in the table. How was Cal calculating the sale price? Explain your answer using sample calculations.

6. Check some of G.D.'s other charges in the table. How was G.D. calculating the sale price? Explain your answer using sample calculations.

7. Who do you think was right, and why?