Powers, Grade 8— Teacher Notes

Philosophy

The basic philosophy of these lessons is to *teach for understanding*. Thus:

- The lessons start by describing a situation *without invoking new vocabulary or notation*. This could be a simplified real world context, or just a number pattern. The main point is not so much that they will necessarily discover new concepts, but that they will be better prepared to hear a teacher explanation. If some of them discover the concepts, all the better.
- The particular topics of percent and powers both require *extensive use of the calculator*. This is consistent with the Common Core emphasis on smart use of tools. See the note below on possible exceptions to this.
- Once the concept and any related techniques are introduced, students apply them to other problems.
- Some of the problems involve making a table of change over time. Other problems involve trial and error (guess and check, if you prefer.) These kinds of problems are a form of *practice* that is not boring, or at any rate, less boring than random exercises.
- There is a fair amount of *repetition*, because the ideas are difficult, but hopefully not exact repetition.
- Because some of the work is challenging, allow the students to help each other, and be ready to help them as well, sometimes individually, sometimes through a class discussion.

Once new rules have been learned, post them prominently in the classroom. This way the focus will be on understanding, not memorization. You may at some later date decide to ask for memorization and take them down, but starting with memorization is a recipe for disaster. The kids who can do it feel they can turn off their brain and need not understand what they're doing. This sort of mastery is fragile and does not last. And the kids who have trouble memorizing things they don't understand are frozen out of the lesson altogether. Neither group benefits if you give up on teaching for understanding.

No-Calculator Activities

While the calculator is assumed throughout these lessons, you can and should do some no-calculator activities along the way. For example you can ask the students to figure out mentally or on paper basic power questions, like:

- What is 3^3? 10^3? (you can do a bunch of the easiest ones before you get fancier)
- What is 2^3? What is $2^3 \cdot 2^3$? What is $(2^3)^2$?
- etc.

The purpose of the mental math is two-fold: on the one hand, it shows that one need not run to the calculator every time; on the other hand, it helps kids focus on the meaning of powers in a way that just working with the calculator cannot do.

CCSS-M Alignment

This series of lesson on Exponents is designed to prepare the students for the following Grade 8 standard:

8.EE 1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \cdot 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$.

Because these materials are designed to foster teaching for understanding they naturally hit on other CCSS standards for Grade 8 (for example, other aspects of 8. F.) They also revisit Grade 6 (6.EE 9) and prepare students for 8.EE 2 and Algebra 1 (N-RN).

Lesson 1: Bacteria

Before starting this lesson, write pairs of numbers on the board, such as
    5   15
For each pair, ask these two questions:
    15 is how much more than 5? (Answer: 10)
    15 is how many times as much as 5 (Answer: 3)
Once they are clear on the meaning of the questions, start the lesson.

#1 helps motivate what follows. Students should make guesses. If they try to actually make calculations, you can point out they started #2.

On the one hand, the lesson is a review of the basic meaning of whole-number exponents. On the other hand, it brings attention to the fact that comparisons involving exponents are easiest when asking "how many times as much?" That understanding is the foundation of much of what is to follow in this unit.

Lesson 2: A Tripling Population

This lesson leads to the basic law of exponents: if the bases are the same, you find the product by adding the exponents. Don't announce this result at the beginning! It is intended to be the final result of working on this sheet. After the students have done the sheet is a good time to make this rule explicit, and post it on the bulletin board: $x^p \cdot x^q = x^{p+q}$

If there is more time available, it is easy to make up problems of this type. Be sure to include problems with a missing factor as in #3.

Lesson 3: Divide and Conquer

This lesson follows from the previous one, by using the fact that division is the inverse operation to multiplication. If students were able to do multiplication problems like #2, they can divide. Again, do not spill the beans prematurely: the rule is the outcome of working on the lesson. At that time, it should be stated, and written on the board or bulletin board: $\dfrac{x^p}{x^q} = x^{p-q}$

Again, if there is time, you can easily put practice problems on the board, overhead, or document reader.

Lesson 4: The Exponent Zero

This lesson uses patterns to explain why any number to the power zero equals 1. You can also get there by using multiplication: $3^2 \cdot 3^0 = 3^{2+0} = 3^2$. The only way that could work is if $3^0 = 1$. Or by using division: $\frac{3^2}{3^2} = 3^{2-2} = 3^0$. Again, the only way that could work is if $3^0 = 1$.

Lesson 5: Equal Powers

This lesson involves a lot of calculator work. The idea is for students to get comfortable with exponents, and to lay the groundwork for the power of a power rule, which will be introduced in the next lesson.

Lesson 6: Using Different Bases

The lesson starts by applying the results from Lesson 5, and ends with the power of a power rule.

#5 will almost certainly require a whole-class discussion. Ask students to give examples of writing numbers as powers in different ways. At first, allow examples from the worksheet, but encourage students to come up with other examples. Finally, challenge them with finding examples that can only be done one way.

Again, post the rule on the bulletin board!

Lesson 1: Bacteria

1. A colony of bacteria is being grown in a laboratory. It contains a single bacterium at 12:00 noon *(time* 0), and the population is doubling every hour. How long do you think it would take for the population to exceed I million? 2 million? Write down your *guesses* and compare them with other students' guesses.

2. Make a table of values showing how the population in problem 1 changes as a function of time. Find the population one hour from now, two hours from now, etc. Extend your table until you can answer the questions asked in problem 1. How close were your guesses?

Time (hours)	**Bacteria**	**Powers of 2**
0	1	
1	2	
2	4	

3. In the third column, write the population each time as a power of 2.

4. What would the population be after x hours? (Write this as a power of 2.)

Making Comparisons

Let us compare the populations at different times.

5. Compare the population after 8 hours with the population after 5 hours.
   a. *How much more* is the population after 8 hours? (Compare by subtracting.)

   b. *How many times* as much is it? (Compare by dividing.)

   c. Which of your answers in (a) and (b) can be written as a power of 2? What power of 2 is it?

6. Repeat problem 5, comparing the population after 7 hours with the population after 3 hours.

7. One of the questions, *How much more than?* or, *How many times as much?* can be answered easily with the help of powers of 2. Which question? Explain.

8. Make the comparisons below, answering the question: *How many times as much?* Write your answers as powers of 2.
   a. Compare the population after 12 hours with the population after 10 hours.

   b. Compare the population after 9 hours with the population after 4 hours.

   c. Compare the population after 4 hours with the population after 12 hours.

9. Compare each pair of numbers. The larger number is how many times as much as the smaller number? Write your answer as a power.
   a. 2^6 and 2^9

   b. 2^9 and 2^{14}

   c. 2^{14} and 2^6

Lesson 2: A Tripling Population

A colony of bacteria being grown in a laboratory contains a single bacterium at 12:00 noon (time 0). This population is *tripling* every hour.

1. Make the comparisons below, answering the question: How many times as much? Write your answers as powers of 3.
   a. Compare the population after 12 hours with the population after 10 hours.

   b. Compare the population after 9 hours with the population after 4 hours.

   c. Compare the population after 4 hours with the population after 12 hours.

2. Compare each pair of numbers. How many times as much as the smaller number is the larger? Write your answer as a power.
   a. 3^6 and 3^9

   b. 3^9 and 3^{14}

   c. 3^{14} and 3^6

3. By what number would you have to multiply the first power to get the second power? Write your answer as a power.
   a. $3^5 \cdot$ ___ $= 3^{15}$

   b. $3^8 \cdot$ ___ $= 3^{15}$

   c. $3^{11} \cdot$ ___ $= 3^{15}$

Multiplying Powers

Raising to a power is another way to say "repeated multiplication." The exponent tells how many times the base has been used as a factor. For example, 4^2 means $4 \cdot 4$, and 4^3 means $4 \cdot 4 \cdot 4$, therefore:

$4^2 \cdot 4^3 = (4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) = 4^5$

Use this idea to multiply powers.

4. Write the product as a power of 3.
   a. $3^7 \cdot 3^3 =$

   b. $3^5 \cdot 3^5 =$

   c. $3^8 \cdot 3^2 =$

5. Write the product as a power of 5.
   a. $5^4 \cdot 5^3 =$

   b. $5^4 \cdot 5^6 =$

   c. $5^4 \cdot 5^9 =$

6. Write the product as a power. Careful!
   a. $5^a \cdot 5^b =$

   b. $x^2 \cdot x^3 =$

   c. $x^p \cdot x^q =$

Lesson 3: Divide and Conquer

When we divide, the quotient tells us how many times as much the numerator is than the denominator. For example, 6/3 means “what times 3 equals 6?” Since the answer is 2, we have 6/3 = 2

Let us apply this idea to dividing powers:

$4^5 / 4^2$ means “what times 4^2 equals 4^5?” Since $4^2 \cdot 4^3 = 4^5$, you have $4^5 / 4^2 = 4^3$.

Let us start by reviewing multiplying powers:

1. Write the product as a power of 2.
   a. $2^6 \cdot 2^5 =$

   b. $2^3 \cdot 2^3 =$

   c. $2^3 \cdot 2^8 =$

2. Fill in the blanks:
   a. $2^3 \cdot$ ___ $= 2^6$

   b. $2^3 \cdot$ ___ $= 2^{11}$

   c. $2^6 \cdot$ ___ $= 22^{11}$

Now we're ready to divide powers.

3. Write the quotient as a power of 2.

a. $\frac{2^{11}}{2^{6}} =$

b. $\frac{2^{11}}{2^{5}} =$

c. $\frac{2^{11}}{2^{3}} =$

4. Write the quotient as a power of 3.

a. $\frac{3^{7}}{3^{5}} =$

b. $\frac{3^{6}}{3^{4}} =$

c. $\frac{3^{99}}{3^{33}} =$

Definitions: In the expression 2^9, 2 is the *base*, and 9 is the *exponent*.

5. In all the exercises in this lesson, the bases are the same. Put the patterns you discovered into words.
   a. When multiplying two powers with the same base, what is the base of the product? What is the exponent?

   b. When dividing two powers with the same base, what is the base of the quotient? What is the exponent?

Lesson 4: The Exponent Zero

A wizard starts with 1 gold piece on December 31. Every day, his magic multiplies the number of gold pieces by 4. He kept track of his gold pieces in a table:

Date	Days	Treasure	Powers of 4
December 31		1	
January 1	1	4	4^1
January 2	2	16	4^2
January 3	3		

1. Fill out the empty cells in the table.

2. Give a number for each expression. Hint: the table may help you.
   a. $4^3 =$

   b. $4^0 =$

3. Here is a table of powers of 5. Fill out the empty cells.

5^5	3125
5^4	625
5^3	
5^2	
5^1	
5^0	

4. Study the table of powers of 5. As you go down the table, how do you get the next
   a. Power of 5?

   b. Number?

5. Looking at the patterns in the tables, do you agree or disagree that $4^0 = 5^0$?

6. What do you think 2^0 is?

7. Some students think that $3^0 = 0$. They are wrong. Explain to them why they are wrong.

8. Any number raised to the power 0 equals ____. Or, to put it another way: $x^0 =$ ____

This is surprising, but true!

Lesson 5: Equal Powers

In this lesson, use only whole number exponents.

1. **Exploration**: 64 can be written as a power in at least three different ways: 2^6, 8^2, or 4^3.
   a. Find some numbers that can be written as powers in two different ways.

   b. Find another number that can be written as a power in three different ways.

Powers of 3 and 9

2. Using your calculator if necessary, try to find a power of 3 that is equal to each power of 9. If any are impossible, say so. Fill in the exponent.

   a. $9^2 = 81 = 3^?$
   b. $9^5 = 59049 = 3^?$
   c. $9^{10} = 3^?$
   d. $9^0 = 3^?$

3. Using your calculator if necessary, try to find a power of 9 that is equal to each power of 3 below. If any are impossible, say so. Fill in the exponent.

   a. $3^8 = 6561 = 9^?$
   b. $3^5 = 243 = 9^?$
   c. $3^{14} = 9^?$
   d. $3^0 = 9^?$

4. **Summary**:
   a. Can every power of 9 be written as a power of 3? If so, explain why. If not, show some that can and some that can't, and explain the difference.

   b. Can every power of 3 be written as a power of 9? If so, explain why. If not, show some that can and some that can't, and explain the difference.

Powers of 2, 4, 6, and 8

5. Find two powers of 2 (other than 64) that can be written as powers of 8.

6. If the same number is written as both a power of 2 and a power of 8, how do the exponents compare? Explain and give examples.

7. Find at least three powers of 2 that can be written as powers of 4. Compare the exponents and describe what you notice.

8. Find at least two powers of 2 that can be written as powers of 16. Compare the exponents and describe what you notice.

9. a. Which powers of 2 can be written as powers of 8? Explain, giving examples.

   b. Which powers of 8 can be written as powers of 2? Explain, giving examples.

   c. Find the smallest number (besides 1) that can be written as a power of 2, a power of 4, and a power of 8. Write it in all three ways. How do you know that it is the smallest?

10. Can you find a number that can be written as a power of 2, a power of 4, and a power of 6? If so, find it. If not, explain why it is impossible.

Lesson 6: Using Different Bases

1. Write each number as a power using a smaller base.

   a. 8^2 b. 27^3 c. 25^3

   d. 16^4 e. 49^2 f. 2^0

2. Write each number as a power using a larger base.

   a. 3^2 b. 9^4 c. 4^8

   d. 5^8 e. 6^6 f. 95^0

3. If possible, write each number as a power using a different base. (Do not use the exponent 1.) If it is not possible, explain why not.

   a. 3^4 b. 3^3

   c. 4^5 d. 3^5

4. Repeat problem 3 for these numbers.

   a. 5^4 b. 5^3

   c. 25^2 d. 26^4

5. **Summary**: If you exclude the exponent 1, when is it possible to write a number in two or more ways as a power? Does it depend on the base, the exponent, or both? Explain. (Give examples of some equivalent powers and of numbers that can be written as powers in only one way.)

6. **Generalization**: Fill in the exponents.

   a. $9^x = 3^?$ b. $8^x = 2^?$

   c. $16^x =$ d. $25^x =$

A Power of a Power

Since $9 = 3^2$, the power 9^3 can be written as $(3^2)^3$. The expression $(3^2)^3$ is a *power of a power* of 3.

7. a. Write 25^3 as a power of a power of 5.

   b. Write 8^5 as a power of a power of 2.

   c. Write 9^4 as a power of a power of 3.

There is often a simpler way to write a power of a power. For example:

$$\begin{aligned} (3^5)^2 &= (3^5)(3^5) \\ &= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \\ &= 3^{10} \end{aligned}$$

8. a. Show how $(2^5)^3$ can be written with one exponent as a power of 2.

   b. Write $(3^4)^2$ as a power of 3.

9. Is $(4^5)^3$ equal to 4^8 or 4^{15} or neither? Explain.

10. **Generalization**: Fill in the exponents.

a. $(x^2)^3 = x^?$

b. $y^4 = (y^2)^?$

c. $y^{10} = (y^?)^5$

d. $(x^4)^3 = x^?$

11. **Generalization**: Fill in the exponents.

a. $(y^2)^x = y^?$

b. $(y^3)^x = y^?$

c. $(x^4)^y = x^?$

d. $y^{ax} = (y^x)^?$

The generalization you made is one of the *laws of exponents*.
It is sometimes called the *power of a power law*:

$(x^a)^b = x^{ab}$, as long as x is not 0.