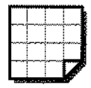
PERIMETER AND AREA PATTERNS

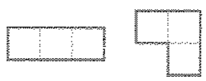
1.1 Polyominoes

You will need:

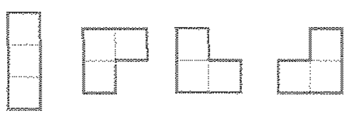
graph paper 

Definition: *Polyominoes* are shapes that are made by joining squares edge-to-edge. The best known example is the *domino.*

Using three squares, you can find two different *trominoes*, the straight one and the bent one.



There are only two trominoes. The bent one is shown in different positions.



The following shapes are *not* polyominoes.

1. What part of the definition do they violate?

DISCOVERING POLYOMINOES

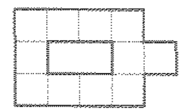
2. *Tetrominoes* are made up of four squares. There are five different tetrominoes. Find all of them.

3. Guess how many squares make up a *pentomino*. Find all twelve pentominoes. Make sure you do not "find" the same one more than once!

4. Find as many *hexominoes* as you can.

AREA AND PERIMETER

Large polyominoes may have holes in them, as in this 11-omino (i.e. polyomino of area 11).



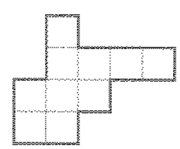
In this book, we will not discuss polyominoes with holes.

Definitions: The *area* of a two-dimensional figure is the number of unit squares it would take to cover it. The *perimeter* of a figure is the distance around it.

For example, the area of the domino is 2, and its perimeter is 6.

In this book, area and perimeter will provide you with many opportunities to discover and apply algebra concepts.

5. Here is a 10-omino. What is its area? What is its perimeter?



6. Draw some 10-ominoes, and find the perimeter of each one. It would take too long to find all of the 10-ominoes, but try to find every possible 10-omino perimeter.

7. Repeat problem 6 for 16-ominoes.

8. Draw as many polyominoes as you can having 10 units of perimeter, and find the area of each one.

9. Find some polyominoes having perimeter 16. It would take too long to find all of them, but try to find every possible area.

10. **Summary** Describe any patterns you noticed when working on this lesson.

11. Have you found any polyominoes having an odd-number perimeter? If you have, check your work. If you haven't, explain why.

Area and perimeter of polyominoes are related. It is not a simple relationship: for a given area, there may be more than one perimeter possible. For a given perimeter, there may be more than one area.

12. **Project** The words *polyomino, tetromino, pentomino, hexomino* all end the same way, but they start with different prefixes.
   a. Find other words (not just from mathematics) that start with the prefixes *poly-, tetr-, pent-,* and *hex-*. Tell the meaning of each word.
   b. What are the prefixes for 7, 8, 9, and 10? Find words that begin with those prefixes. Tell the meaning of each word.
   c. Write a story using as many of the words you found as possible.

PREVIEW DIMENSIONS

- The following are one-dimensional: a line, the boundary of a soccer field.
- The following are two-dimensional: the surface of a lake, the paper wrapped around a present.
- The following are three-dimensional: an apple, a person.

An object like a sheet, while it does have some thickness and therefore is three-dimensional, can be thought of as a model of a two-dimensional surface with no thickness. Similarly, a wire or even a pencil can be thought of as a model of a one-dimensional line.

13. Divide the following into three groups: one-, two-, or three-dimensional.
   a. a book
   b. a lake
   c. a map
   d. a piece of paper
   e. a piece of string
   f. an algebra student
   g. Mickey Mouse
   h. the boundary of a county
   i. the water in a glass
   j. the paint on a house

14. Name three objects of each kind.
   a. one-dimensional
   b. two-dimensional
   c. three-dimensional

Getting comfortable with the concept of dimension will help you with some of the algebra concepts that you will study later in this course.

15. Draw a picture that incorporates several of your objects of different dimensions.

16. Write a short paragraph explaining what 3-D glasses are used for.

1.2 Perimeter of Polyominoes

You will need:

graph paper

SHORTEST AND LONGEST PERIMETER

For polyominoes with a given area, there may be more than one perimeter. In this section, you will try to find the shortest and the longest perimeter for each given area.

1. Copy this table, extend it to area 24, and fill it out. (A few rows have been done for you.) Experiment on graph paper as much as you need to, and look for patterns.

	Perimeter	
Area	**Shortest**	**Longest**
1	4	4
2	6	6
3		
4	8	10
5		
...		

2. What patterns do you notice in the table? Explain.

3. Describe the pattern for the perimeter of a polyomino of area *A*, having:

a. the longest perimeter;

b. the shortest perimeter.

4. For a polyomino having a given area, what perimeters are possible between the shortest and longest? (For example, for area 4, the minimum perimeter is 8, and the maximum is 10. Is it possible to have a perimeter of 9?)

5. What perimeters are possible for area 9?

MAKING PREDICTIONS

Mathematics is the science of patterns. Discovering a pattern can help you make predictions.

6. Predict the longest possible perimeters for polyominoes having these areas. If the number is not too big, experiment on graph paper to test your predictions.

a. 36 b. 40 c. 100
d. 99 e. 101 f. 1000

7. Explain your method for answering problem 6.

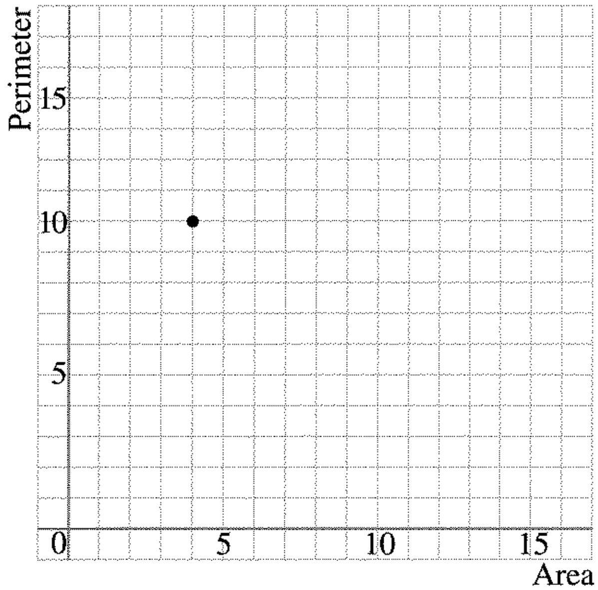
8. Predict the shortest possible perimeters for polyominoes having these areas. If the number is not too big, experiment on graph paper to test your predictions.

a. 36 b. 40 c. 100
d. 99 e. 101 f. 1000

9. Explain your method for answering problem 8.

MAKING A GRAPH

10. On graph paper, draw a horizontal axis and a vertical axis. Label the horizontal axis *Area* and the vertical axis *Perimeter,* as in the following graph. Extend them as far as you can, to at least 25 units for area and 55 units for perimeter.



Definition: The point where the axes meet is called the *origin.*

For the following problems, you will need the numbers you found in the table in problem 1.

11. For each area, there is one number for the longest perimeter. For example, the longest perimeter for an area of 4 is 10. This gives us the number pair (4, 10). Put a dot on the graph at the corresponding point. (Count 4 spaces to the right of the origin, and 10 spaces up.) Do this for all the *area and longest perimeter* points on the table.

12. Describe what the graph looks like.

13. Using the same axes, repeat problem 1 with the numbers for area and shortest perimeter. One dot would be at (4, 8).

14. Describe what the graph looks like.

INTERPRETING THE GRAPH

15. Explain why the first set of points is higher on the graph than the second set.

16. As the area grows, which grows faster, the longest perimeter or the shortest perimeter? What happens to the gap between the two?

17. Use the graph to figure out how many different perimeters are possible for an area of 25. Explain how you did it.

18. Use the table you made in problem 1 to answer problem 17. Explain how you did it.

19. Use the graph to check whether there is a polyomino having area 15 units and perimeter 20. Explain how you did it.

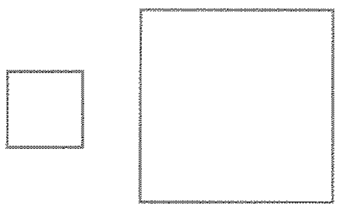
20. Use the table you made in problem 1 to answer problem 19. Explain how you did it.

In this lesson you used patterns, tables, and graphs to help you think about a problem. This is an important skill which you will develop throughout this course.

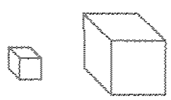
PREVIEW UNITS AND DIMENSIONS

Length is measured in linear units, such as the inch (in.) or centimeter (cm). Length refers to one dimension.

Area is measured in square units, such as the square inch ($in.^2$, or sq in.) or square centimeter (cm^2). Area refers to two dimensions.



Volume is measured in cubic units, such as the cubic inch ($in.^3$, or cu in.) or cubic centimeter (cm^3, or cc). Volume refers to three dimensions.



21. Divide the following units into three groups according to what they measure: length, area, or volume.

a. acre
b. fluid ounce
c. foot
d. gallon
e. kilometer
f. liter
g. meter
h. mile
i. pint
j. quart
k. yard

22. For each unit listed in problem 21, name something that might be measured with it. For example, for (a), the area of a farm could be measured in acres.

LESSON 1.3

Introduction to the Lab Gear

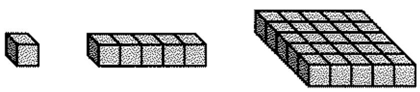
You will need:

the Lab Gear 

The Lab Gear blocks come in two colors, yellow and blue.

THE YELLOW BLOCKS

The yellow blocks represent whole numbers, such as 1, 5, or 25.



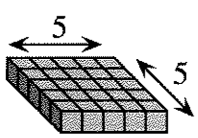
1. Use the Lab Gear to represent these quantities. Write down what blocks you used.

a. 13 b. 21

2. Find as many different numbers as possible that can be represented by using exactly three yellow blocks.

3. Write some numbers that *cannot* be represented by the Lab Gear. Explain why you believe this to be true.

You will soon learn to use the Lab Gear for negative numbers. Later, you will use the Lab Gear to work with fractions.



Notice that the block that represents 25 is a 5-by-5 square.

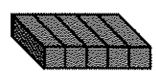
Notation: In algebra, the multiplication 5 times 5 is written $5 \cdot 5 = 25$, or $5(5) = 25$. Do not use x to indicate multiplication—it could be confused with the letter x. When handwriting, use a dot, and when typing or using a computer, use an asterisk: 5 * 5 = 25. In this book, we will use the dot.

THE BLUE BLOCKS

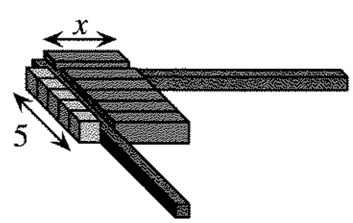
The blue blocks represent *variables.*
All the Lab Gear variables are related to these two blocks.

Variables are usually named by letters. Since the names x and y are used most often in algebra, they have been chosen to name the variables in the Lab Gear.

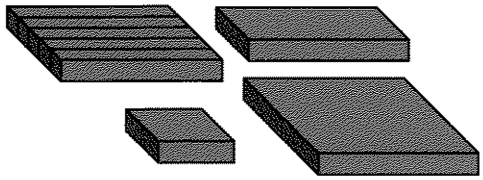
4. Write a way to remember which block is x and which block is y.



This block represents $5 \cdot x$ (which is usually written as $5x$). The reason it is $5x$ can be seen by counting the number of x's that make it. Another way to see it is to notice that it is a rectangle. In a rectangle, the area is equal to the length times the width. Using the corner piece, we can measure the $5x$ block, and see that its dimensions are 5 and x, and its area is $5x$ square units.



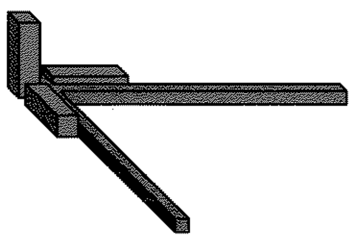
5. Using the corner piece, find the measurements of each of these blocks in terms of x and y. Sketch each block. Label each one with its dimensions and area.



Notation: In algebra, $5 \cdot x$ is written $5x$, and $x \cdot y$ is written xy. (When no operation is indicated, multiplication is understood.) $x \cdot x$ is abbreviated x^2, and read *x squared*, or *the square of x*.

6. Explain why x^2 is read *the square of x.*

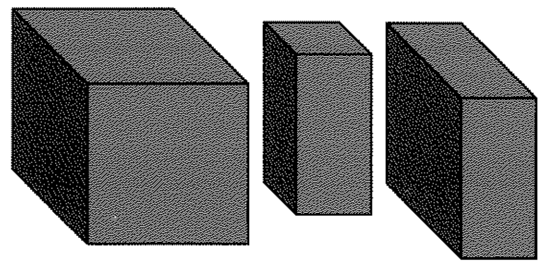
The following figure shows $x \cdot x \cdot x$ in the corner piece. There is a block whose measurements in three dimensions (length, width, height) match those shown.



7. Which block would fit in the corner piece with those measurements? What shape is it?

8. In algebra the quantity $x \cdot x \cdot x$ is read *x cubed*, or *the cube of x*. Why do you think it is called that?

9. Use the corner piece to find the length, width, and height of each of the remaining blocks in terms of x and y.



10. **Summary** Sketch each Lab Gear block, and label it with its name. Keep these labeled sketches in your notebook for future reference. (However, if you forget the name of a block, you don't need to look it up. Just measure it, using the corner piece.)

11. Sketch what each of the following would look like with the Lab Gear. If an expression is impossible to show with the Lab Gear, explain why.
    a. $x^2 + x^2 + 3$
    b. $x^2y + xy$
    c. $x + x^2 + x^3$
    d. $x^3 + x^4$

DISCOVERY ***HANDSHAKES***

12. There are nine teachers at a math department meeting. They decide to shake hands with each other before starting the meeting. Each teacher is to shake hands exactly once with each other teacher. How many handshakes does it take? Explain your answer and how you arrived at it.

(Hint: You may use sketches to help you solve the problem. A good approach is to start out by counting the handshakes if there are two, three, four, five people at the meeting, and by looking for a pattern.)

LESSON 1.4

Variables and Constants

You will need:

the Lab Gear 

Variables are one of the most important concepts of algebra. A variable can stand for different numbers at different times. For example, x could be a positive or a negative number or 0. It could be greater than y, less than y, or equal to y.

Because they do not change, numbers are called *constants.*

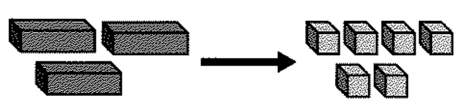
SUBSTITUTING

Definition: Replacing a variable by a constant amount is called *substitution.*

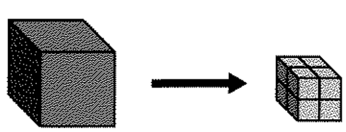
Example: The figure shows how the Lab Gear can be used to show the substitution $x = 2$ for the expressions x, $x + 2$, $3x$, x^2, and x^3.

x: 

$x + 2$: 

$3x$: 

x^2: 

x^3: 

1. Sketch what $x + 2$ looks like modeled with the Lab Gear. Then sketch what it looks like if the following substitution is done.
   a. $x = 5$ b. $x = 1$
   c. $x = 3$ d. $x = 0$
2. Repeat problem 1 for x^2.
3. Repeat problem 1 for x^3.
4. Repeat problem 1 for $3x$.

An expression that involves x can have many different values, depending on the value of x.

EVALUATING

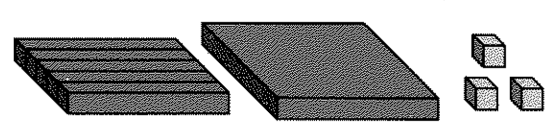
Definition: To *evaluate* an expression means to find its value for a particular value of x.

Looking back at the figure in the previous section, you can see the value of each expression when $x = 2$. The figure shows that $x + 2 = 4$, $3x = 6$, $x^2 = 4$, and $x^3 = 8$.

In the following problems:

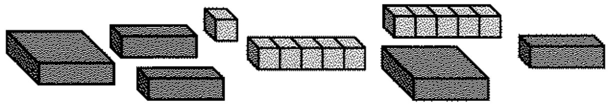
- Put out blocks to match each figure.
- Replace the variables (represented by blue blocks) with the given constants (represented by yellow blocks).
- Evaluate each expression by counting what you have.

5. Evaluate for :
   a. $y = 1$; b. $y = 2$; c. $y = 0$.



6. Evaluate for :

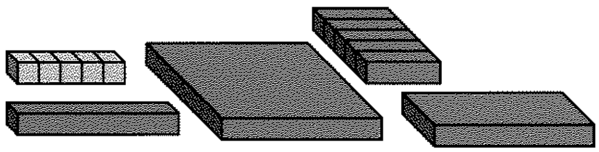
a. $x = 1$; b. $x = 5$; c. $x = 0$.



7. Evaluate for :

a. $x = 5$ and $y = 4$

b. $x = 4$ and $y = 0$



Evaluate these expressions without using the Lab Gear. You may want to use your calculator.

8. $y^2 + 5y + 3$ if $y = 1.3$

9. $y^2 + xy + 5x + y + 5$ if $x = \frac{1}{2}$ and $y = 4$

Evaluating expressions is important in many walks of life, from science and engineering to business and finance. It is usually done with the help of calculators and computers. In this course you will learn some of the ideas that are built into calculators and computers.

FINDING X

Use trial and error for these problems.

10. If $x + 2 = 18$, what is x?

11. What is x if

a. $3x = 18$? b. $x^2 = 64$?

c. $x^3 = 64$?

In a sense, finding x is the reverse of substituting. In future chapters you will learn many methods for finding the value of a variable.

THE SUBSTITUTION RULE

In the following equations, there are two place-holders, a diamond and a triangle. The **substitution rule** is that, within one expression or equation, the same number is placed in all the diamonds, and the same number is placed in all the triangles. (The number in the diamonds may or may not equal the number in the triangles.)

For example, in the equation

$$\Diamond + \Diamond + \Diamond + \Delta = \Delta + \Delta$$

if you place 2 in the $\Diamond$ and 3 in the Δ, you get

$$2 + 2 + 2 + 3 = 3 + 3.$$

Note that even though the diamond and triangle were replaced according to the rule, the resulting equation is *not* true.

12. Exploration The equation

$$\Diamond + \Diamond + \Diamond + \Delta = \Delta + \Delta$$

is not true with 2 in the $\Diamond$ and 3 in the Δ. Find as many pairs of numbers as possible that can be put in the $\Diamond$ and in the Δ to make the equation true. For example, 0 in both the Δ and $\Diamond$ makes it true. Arrange your answers in a table like this:

$\Diamond$	Δ
0	0
...	...

Describe any pattern you notice. Explain why the pattern holds.

For the following equations, experiment with various numbers for $\Diamond$ and Δ. (Remember the substitution rule.) For each equation, try to give three examples of values that make it true. If you can give only one, or none, explain why.

13. $\Diamond + \Diamond + \Diamond = 3 \cdot \Diamond$

14. $\Diamond + \Diamond + \Diamond = 4 \cdot \Diamond$

15. $\Delta + \Delta + \Delta = 3 \cdot \Delta$

16. $\Diamond + \Diamond + 2 = 3 \cdot \Diamond$

17. $\Diamond + \Diamond + 2 = 2 \cdot \Diamond$

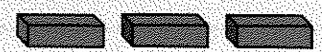
18. $\Diamond \cdot \Delta = \Delta \cdot \Diamond$

19. $\Diamond \cdot \Delta = \Delta + \Diamond$

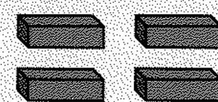
20. $\Diamond \cdot \Diamond \cdot \Diamond = 3 \cdot \Diamond$

21. $\Diamond \cdot \Diamond \cdot \Delta = \Diamond + \Diamond + \Delta$

22. **Report** Say that $\Diamond$ is x and Δ is y. For each equation above, show both sides with a sketch of Lab Gear blocks. In some cases, the sketches may help you explain whether the equations are always true or not. For example, for problem 13 both sides would look like this.



But, for problem 14 the right side would look like this. Write an illustrated report about what you did.



THINKING WRITING

1.A Graphing Rectangle Areas

How does the area of a rectangle change if you vary either the length or the width and leave the other dimension unchanged? How does the area of a rectangle change if you vary both the length and the width? Tables and graphs will help you investigate these questions and notice patterns.

1. What is the area of a rectangle having the following dimensions?

a. 1 by 9 b. 2 by 9
c. 3 by 9 d. 9 by 9

2. What is the area of a rectangle having the following dimensions, if $x = 10$?

a. 1 by x b. 2 by x
c. 3 by x d. x by x

3. Make a table like this, extending it to $x = 6$.

	Area of rectangle having dimensions:			
x	1 by x	2 by x	3 by x	x by x
1	1	2	3	1
2	...	...	...	...

4. Draw axes, with x on the horizontal axis, and area on the vertical axis. Plot the points you obtained in problem 3 for the area of 1-by-x rectangles. For example, (1, 1) will be on the graph.

5. Does it make sense to connect the points you plotted? What would be the meaning of points on the line, in between the ones you got from your table? Label your graph *1 by x.*

6. On the same axes, graph the data you obtained for 2-by-x, 3-by-x, and x-by-x rectangles. For more accuracy on the last one, you may use your calculator to find points for $x = 0.5$, 1.5, and so on. Label your graphs *2 by x, 3 by x....*

7. **Report** Write about the four graphs. Describe them and compare them. Your report should reflect what you learned in the above investigation. It should consist of three parts: a problem statement, a detailed explanation, and a conclusion. It should include, but not be limited to, answers to the following questions.

- What is the shape of each graph?
- Which ones are alike? Different? Why?
- How do the first three graphs differ from each other? What is the meaning of that difference?
- What is special about the fourth? Why?
- Do the graphs ever intersect each other? What is the meaning of the points of intersection?
- Where do they cross the vertical axis, and what is the meaning of that point?
- Where does the fourth one cross the others, and what are the meanings of those points?
- Which area grows the fastest? Why?

1.5 Dimensions

You will need:

the Lab Gear 

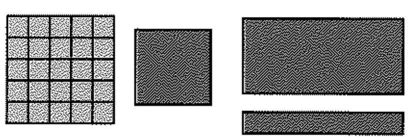
DIMENSIONS AND THE LAB GEAR

Of course, all the Lab Gear blocks are three-dimensional, (as are all objects in the real world). However, we sometimes use the x-block, or the 5-block as a **model** of a one-dimensional object. That is, as a model of a line segment of length x, or 5. Similarly, we can use the x^2- or xy-blocks as models of two-dimensional, flat objects.

1. Some blocks, such as the x^3, cannot be used as models of one- or two-dimensional objects. Make a list of these blocks, which we will call the 3-D blocks.

When making sketches of the Lab Gear, if 3-D blocks or three-dimensional arrangements are not involved, it is much more convenient to work with two-dimensional sketches of the blocks **as seen from above.**

2. Which blocks do these figures represent?



3. Make a 2-D sketch of each of the ten "flat" blocks as seen from above.

4. On your sketch, write *1* on the blocks that model one-dimensional line segments, and *2* on the blocks that model two-dimensional figures.

5. Which block can be thought of as a model of a zero-dimensional point?

6. Sketch the following:
   a. four x-blocks arranged to model a one-dimensional line segment;
   b. four x-blocks arranged to model a two-dimensional rectangle;
   c. four x-blocks arranged to model a three-dimensional box.

7. Sketch the following:
   a. three x^2-blocks arranged to represent a two-dimensional rectangle;
   b. three x^2-blocks arranged to represent a three-dimensional box.

FACES OF THE LAB GEAR

The x^2-block, **as seen from the side**, looks just like the x-block seen from the side, since in either case you see an x-by-1 rectangle.

8. a. Make an x-by-1 rectangle by tracing an x-block.
   b. Place the x^2-block on the rectangle you traced. For it to fit, you will have to stand it on edge.
   c. Which other two blocks can be placed on the rectangle?

9. a. Using a block, trace another rectangle (or square).
   b. Find all the blocks that fit on it.

10. Repeat problem 9, until you have found five more groups of blocks. List each group. Some blocks will appear on more than one list.

In the next sections, when putting blocks next to each other, join them along matching faces.

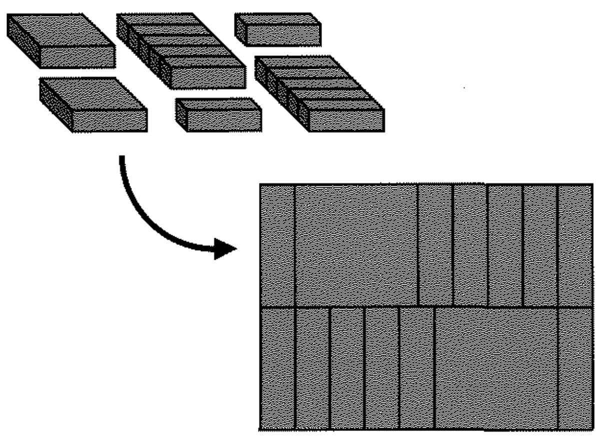
MAKE A RECTANGLE

11. **Exploration** Build each shape and sketch it, showing which blocks you used.
   a. Use only blue blocks; make a rectangle that is not a square.
   b. Use both yellow and blue blocks; make a rectangle that is not a square.
   c. Use both yellow and blue blocks; make a square.
   d. Use only blue blocks; make a square.

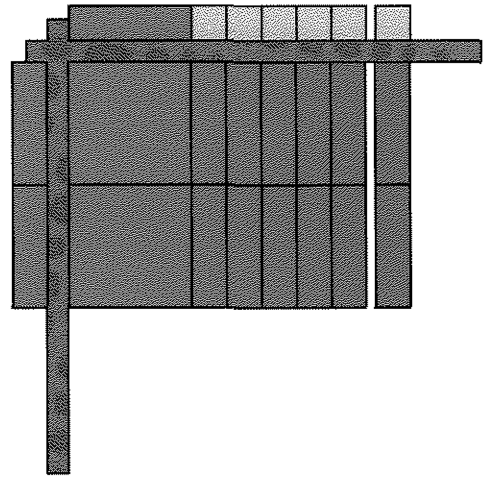
12. Use 1-blocks to make as many different rectangles as you can, having area:
   a. 12 b. 13 c. 14
   d. 30 e. 31 f. 32

13. Make and sketch as many Lab Gear rectangles as you can having area:
   a. $8x$ b. $6xy$

You can rearrange the blocks $2x^2 + 12x$ into a rectangle like this.



The length and width of this rectangle are $x + 6$ and $2x$, which can be seen better if you organize the blocks logically and use the corner piece, as shown. (Notice that you could also turn the rectangle so that the length and width are exchanged. This is considered to be the same rectangle.) The area of the rectangle $2x^2 + 12x$ can be found by just counting the blocks.

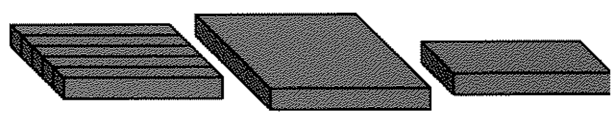


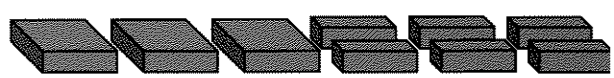
14. There is another rectangular arrangement of the same blocks which has different dimensions. Find it.

For each problem:
   a. Arrange the given blocks into a rectangle in the corner piece.
   b. Sketch it (as seen from above).
   c. Write the length, width, and area.

Find two different solutions for problem 17.

15. 

16. 

17. 

By now you should be able to find the length, width, and area of any Lab Gear rectangle. This will be a useful skill throughout this course.

For each problem, the area of a rectangle is given.

a. Get the blocks that are named.
b. Make the rectangle.
c. Write the length and width.

One problem is impossible. Explain why.

18. $3x^2 + 9x$ **19.** $3xy + 2x + x^2$

20. $4x^2 + 9y$ **21.** $x^2 + 5x$

MAKE A SQUARE

For each problem, the area of a square is given.

a. Get the blocks.
b. Make the square.
c. Write the side length.

One problem is impossible. Explain why.

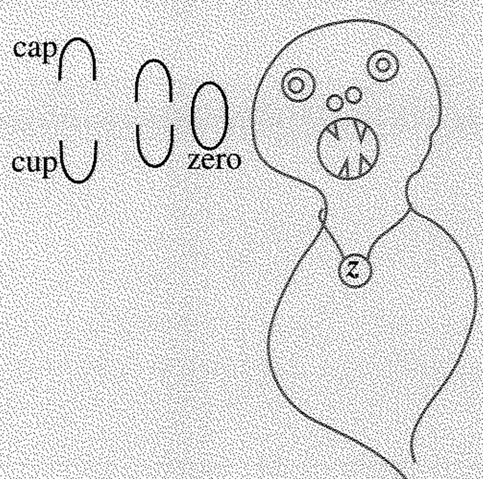
22. 36 **23.** 49

24. 40 **25.** $4x^2$

26. $9x^2$ **27.** $x^2 + 2x + 1$

PREVIEW THE ZERO MONSTER

The Zero Monster eats zeroes. However, all I have to feed it are cups (∪), and caps (∩). It will not eat cups or caps, but it can put one ∪ together with one ∩ to create a zero, which it eats.



For example, if there are three cups and five caps, it will make and eat three zeroes, leaving two caps. This can be written like this:

∪∪∪ + ∩∩∩∩∩ = ∩∩

or like this:

3∪ + 5∩ = 2∩

28. Find out how many zeroes the Zero Monster ate. What was left after it finished eating? Fill in the blanks.

a. 9∪ + 6∩ = ___
b. 9∪ + 6∪ = ___
c. 9∩ + 6∩ = ___
d. 9∩ + 6∪ = ___

29. Fill in the blanks.

a. 4∪ + ___ = 8∩
b. 4∪ + ___ = 8∪
c. 4∩ + ___ = 8∩
d. 4∩ + ___ = 8∪

30. Fill in the blanks.

a. 7∪ + ___ = 1∩
b. 7∪ + ___ = 1∪
c. 7∩ + ___ = 1∩
d. 7∩ + ___ = 1∪

2∩ and 2∪ are examples of *opposites,* because when you add them, you get zero. The concept of opposite is important in algebra, and we will return to it in Chapter 2.

LESSON 1.6 Coming to Terms

You will need:

the Lab Gear 

1. Name a Lab Gear block that can be used as a model for an object with:

a. three dimensions;

b. two dimensions;

c. one dimension;

d. zero dimensions.

Definitions: In the expression

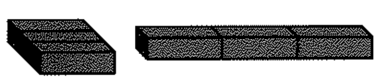
$$x^3 + 2xy - 3x + 4,$$

four quantities are added or subtracted, so we say that there are four *terms:* x^3, $2xy$, $3x$, and 4. Note that a term is a product of numbers and variables. The sum or difference of one or more terms is called a *polynomial.*

Note that polynomials do not involve division by variables. For example, $(1/x) + x$ is not a polynomial.

DEGREE

The *degree* of an expression, in terms of the Lab Gear, is the **lowest dimension** in which you can arrange the blocks. For example, take the expression $3x$. These blocks can be arranged in a rectangle (two dimensions) or in a line (one dimension).



The lowest dimension is one, so the degree of $3x$ is one.

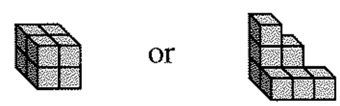
2. Show how the term $2xy$ could be arranged as a box (three dimensions) or as a rectangle (two dimensions). What is the degree of $2xy$?

Of course, x^3 cannot be shown in less than three dimensions, so its degree is 3.

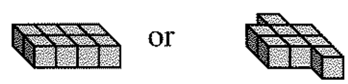
3. Write the degree of each blue block.

The degree of a constant expression (any combination of yellow blocks) is considered to be 0. The reason for this is that the yellow blocks can be separated into 1-blocks, which model zero-dimensional points, with no length, width, or height. See the figure below, which shows how the number 8 can be shown in three ways.

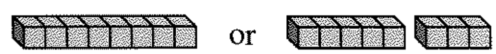
Three dimensions



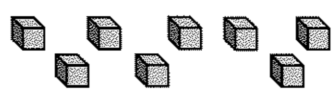
Two dimensions



One dimension



Zero dimensions



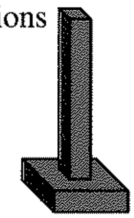
4. What is the degree of these terms?

a. $4y$ b. $5x^2$

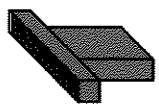
c. $2xy^2$ d. 7

The degree of a polynomial can be found in the same way. For example, the figures below show how the blocks x^2 and y can be arranged in figures of two or three dimensions. However note that they cannot be arranged into figures of zero or one dimension.

Three dimensions



Two dimensions



5. What is the degree of $x^2 + y$?

6. What is the degree of these polynomials?
   a. $4y + 3$ b. $x^3 + 5x^2$
   c. $2xy^2 + x^2$ d. $xy + 7$

Definition: The 2 in the term $2xy$ is called the *coefficient.* A term like x^3 has an invisible coefficient, a 1, since $1x^3$ is usually written just x^3.

7. **Generalizations** If two terms differ only by their coefficients (like $2x$ and $5x$) what can you say about their degrees?

8. How can you find the degree of a term without using the Lab Gear? Explain, using examples.

9. How can you find the degree of a polynomial without using the Lab Gear? Explain, using examples.

HIGHER DEGREE

10. Why is it impossible to show $x^2 \cdot x^2$ with the Lab Gear?

11. What is the product of x^2 and x^2?

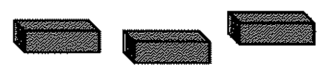
12. Even though there are only three dimensions in space, terms can be of degree 4. Write as many different terms of degree 4 as you can, using 1 for the coefficient and x and y for the variables.

13. Which of these expressions cannot be shown with the blocks? Explain.
    a. $5x^2$ b. $2x^5$ c. $\frac{2}{x^5}$ d. $\frac{5}{x^2}$

COMBINING LIKE TERMS

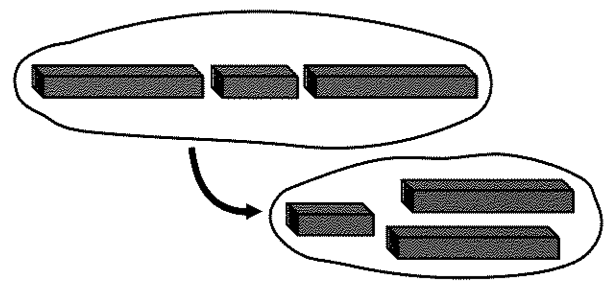
There are many ways you can write an expression that names a collection of Lab Gear blocks. When you put blocks of the same size and shape together and name them according to the arrangement, you are combining *like terms*. Look at these examples.

This quantity is written $x + x + x$,

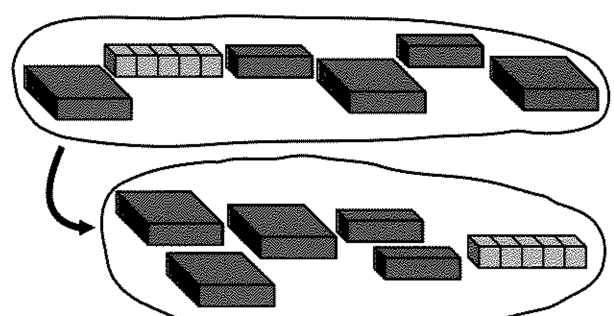


or $3x$, after combining like terms.

This quantity is written $y + x + y$,



or $x + 2y$, after combining like terms.
This quantity is written
$x^2 + 5 + x + x^2 + x + x^2$,

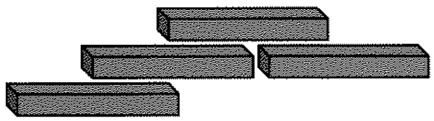


or $3x^2 + 2x + 5$, after combining like terms.

Of course, a $5x$-block, when combining like terms, is equivalent to 5 separate x-blocks. For example, it can be combined with two x-blocks to make $7x$.

For each example, show the figure with your blocks, combine like terms, then write the quantity the short way.

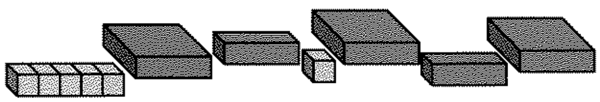
14.



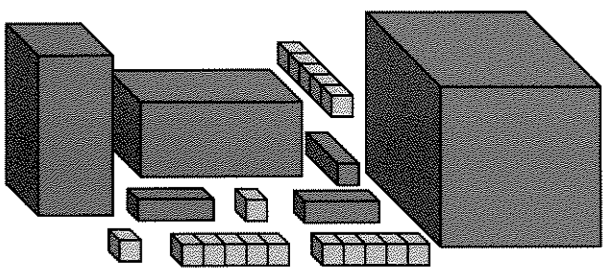
15.

16.

17.



18.



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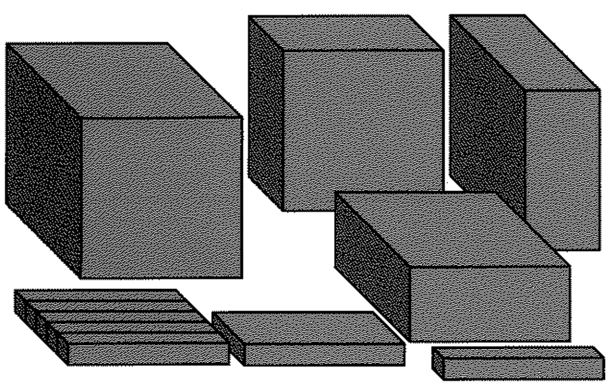
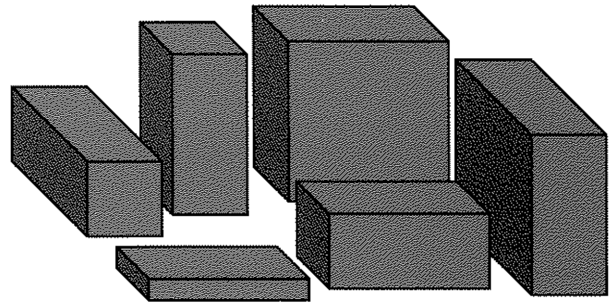


20.



21. What terms are missing? (More than one term is missing in each problem.)

a. $3x^2 + 4x +$ ___ $= 9x^2 + 8x + 7$

b. $x^2y + 6xy +$ ___ $= 9x^2y + 8xy$

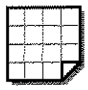
22. **Summary** Explain, with examples, the words *degree, coefficient, polynomial,* and *like terms*. Use sketches of the Lab Gear as well as explanations in words and symbols.

LESSON 1.7

Perimeter

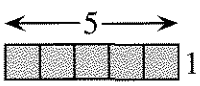
You will need:

the Lab Gear 

graph paper 

PERIMETER OF LAB GEAR BLOCKS

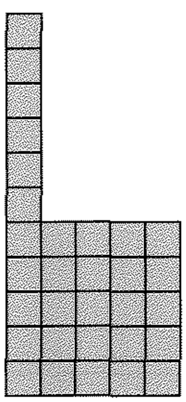
When we discuss the perimeter and area of the Lab Gear blocks, we will be thinking of the tops of the "flat" blocks, which are two-dimensional figures. For example, if you look at the 5-block from above, you would see this figure. Its area is 5 cm^2, and its perimeter is 12 cm.



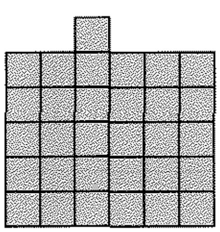
Area: 5 cm^2
Perimeter:
$5 + 1 + 5 + 1 = 12$ cm

Find and write the area and perimeter of these figures, which are the top faces of groups of yellow blocks.

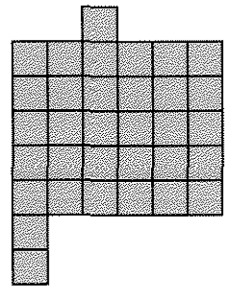
1.



2.



3.

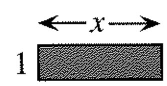


To determine the area and perimeter of the blue blocks, we will not use the actual measurements. Instead, we will consider their dimensions in terms of x and y.

For example, this figure, the top of an x-block, is a 1-by-x rectangle. So its area is x (since $1 \cdot x = x$), and its perimeter is

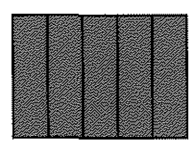
$$x + 1 + x + 1$$

which, by combining like terms, can be written $2x + 2$.

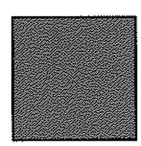


Find and write the area and perimeter of the following rectangles, which are the top faces of blue blocks. Be careful when combining like terms.

4.



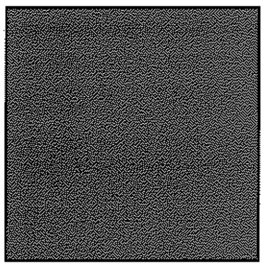
5.



6.

7.

8.



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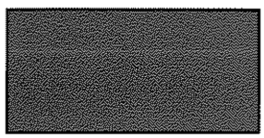
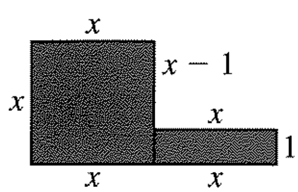


PERIMETER OF LAB GEAR FIGURES

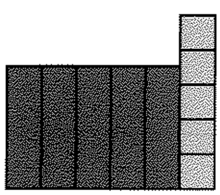
In these problems, assume that x and y are positive. In fact, assume that x is between 1 and 5, and y is between 5 and 10.

Find the perimeter of these figures.

10.



11.



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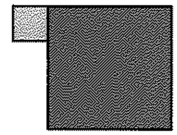

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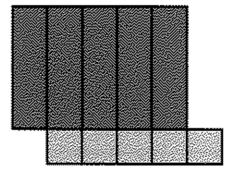

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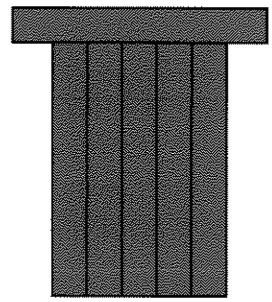

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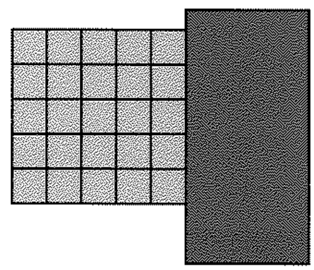


MAKING FIGURES

Use an xy-block and a 5-block to make figures having these perimeters. (These can be any shape. They do not have to be rectangles.) Sketch the figure in each case.

16. $2x + 2y + 2$

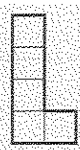
17. $2x + 2y + 10$

18. $2y + 12$

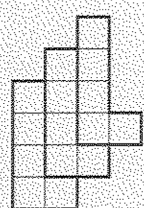
19. Repeat the last three problems using a y-block and a $5x$-block.

20.
a. Use another combination of blocks to get a perimeter of $2x + 2y + 2$.
b. Use another combination of blocks to get a perimeter of $2x + 2y + 10$.
c. Use another combination of blocks to get a perimeter of $2y + 12$.

PENTOMINO STRIPS



21. What is the perimeter of the L pentomino?



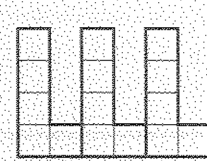
22. Draw a strip of L pentominoes, as shown in the figure above. What is the perimeter if you've used 3 L's?

23. Make a table like this, extending it to 7 rows.

L's	Perimeter
1	...
2	16
3	...

24. Explain how you would find the perimeter of a 100-L strip without drawing it.

25. How many L's were used if the perimeter was 92?



26. Repeat problems 22-25 for an arrangement like the one above.

27. You can use graphs to compare the perimeter patterns for the two pentomino strip arrangements.
a. Draw a pair of axes. Label the horizontal axis *Number of L's* and the vertical axis *Perimeter.*
b. Graph all the number pairs from your first table. For example, since the 2-L strip has a perimeter of 16, you would plot the point (2, 16).
c. On the same pair of axes, graph all the number pairs from your second table.
d. Compare the graphs. How are they the same? How are they different?

28. Repeat problems 22-25 using another pentomino.

POLYOMINO AREA AND PERIMETER

29. Arrange three blocks so that the perimeter of the resulting figure is $6x + 2y$. Find all the solutions you can.

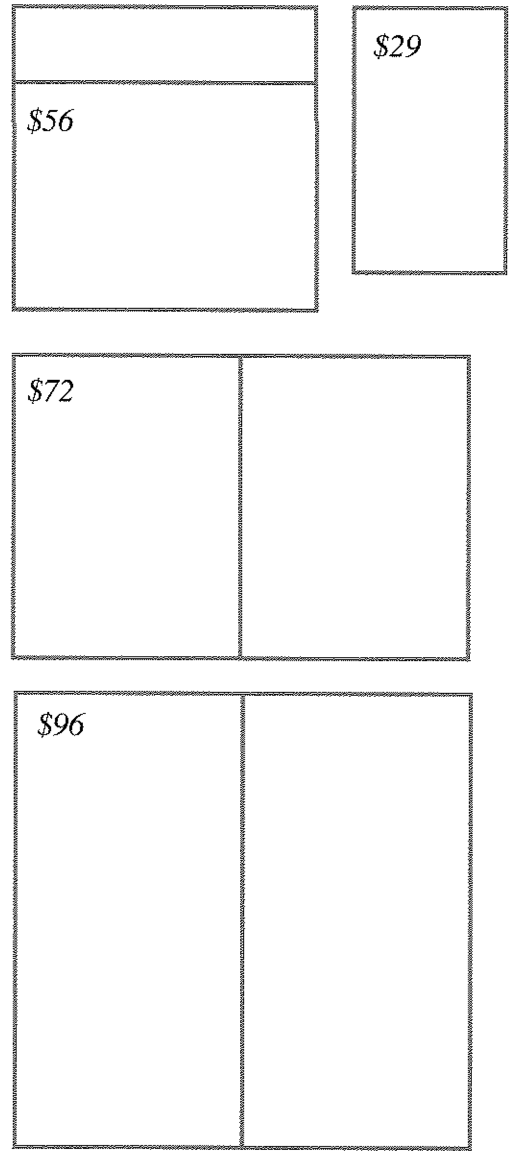
30. Arrange four blocks so that the perimeter of the resulting figure is $8x + 18$. Find all the solutions you can.

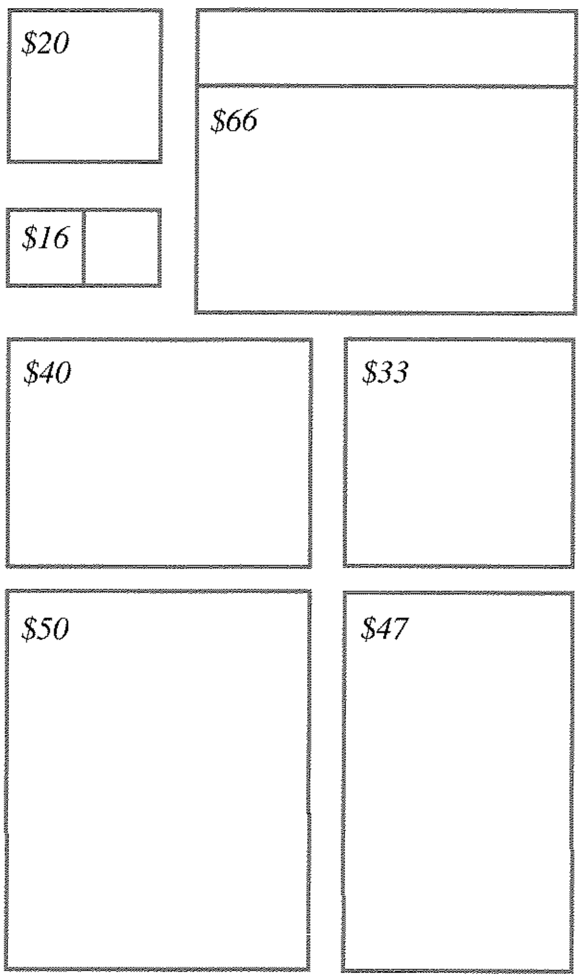
31. Arrange five blocks so that the perimeter of the resulting figure is $2y + 2x + 12$. Find all the solutions you can.

LESSON 1.8

Window-Shopping

On weekends, Lara works at the A.B. GLARE window store. One day a customer, Mr. Alvin Cutterball, asked for an explanation of how the prices were chosen. Lara did not know. Later she asked her supervisor. The supervisor looked very busy and told her not to worry about it. Lara concluded that he probably didn't know and decided she would figure it out herself.





The figures show some of the windows. They are scale drawings, with one centimeter representing one foot.

1. **Exploration** The price of almost all the windows was calculated by following the same principle. Figure out how it was done.

   Important hints:
   - Work with other students.
   - Think about what numbers might affect pricing. (Some possibilities: length, width, area of glass, perimeter of window, number of panes, etc.…)
   - Keep an organized record of the numbers you come across in your exploration.
   - Keep a record of ideas you try, even if they end up not working.

2. One window is on sale and priced below what the system would indicate. Which one is it? How much does it cost when not on sale?

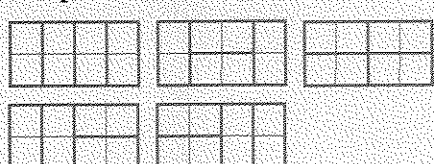
3. Draw scale models of windows that would cost the amounts below. Explain how you got your answers.

   a. $32 b. $84

4. **Report** Summarize your solutions to problems 1, 2, and 3 by writing an explanation of how window prices are calculated. Make it so clear that even Lara's supervisor could understand it. Explain how you figured out the pricing system, showing tables, lists, calculations, diagrams, or anything else that helped you solve the problem.

DISCOVERY *A DOMINO PROBLEM*

The figure shows all the ways to cover a two-by-four strip with dominoes.



5. Find out how many ways there are to cover a two-by-five strip with dominoes. Sketch each way, making sure that you do not show the same way more than once.

6. Make a table like this one about strips of width 2, extending it to length 8. Note that there is only one way to cover a strip of length zero, and that is not to cover it!

Length of strip	Number of ways to cover it with dominoes
0	1
1	1
2	...
3	...
4	5
5	...

7. Look for the pattern in the numbers in the second column. Use the pattern to extend the table to length 10.

THINKING WRITING

1.B Drapes

The A.B. GLARE window store also sells drapes. They stock full-length drapes that go down to the floor, as well as window-length drapes that just cover the window.

One day a customer, Ms. Phoebe Tall, came in with a list of the windows for which she needed drapes.

Window-length drapes:
three 2-by-3-ft windows
two 3-by-3-ft windows

Full-length drapes:
two 3-by-7-ft door-windows

Undecided:
four 3-by-4-ft windows
six 3-by-5-ft windows

(The second number represents the height.)

The material Ms. Tall selected is priced at $3 per square foot. All her windows (except the door-windows, of course) are 3 feet above the floor.

She asked Lara to help her figure out what the cost of various options would be. She wanted to know the smallest amount she could spend. She also wanted to know how much it would cost if she used full-length drapes for all the "undecided" windows. After listening to Lara's explanations, she revealed that she was planning to spend no more than $800. Figure out what Lara should advise her to do.

1. Figure out the smallest amount Ms. Tall could spend, assuming that all the undecided windows are covered with window-length drapes. (Hint: First find the total area. Drawing sketches might help.)
2. Figure out the largest amount she could spend, assuming that all the undecided windows are covered with full-length drapes.
3. If she were planning to spend no more than $800, how many of the undecided windows could she cover with full-length drapes?
4. **Report** Write a full explanation of the results of your investigation. Include sketches that Lara could use to explain the options to Ms. Tall. Your report should consist of three parts: a problem statement, a detailed explanation, and a conclusion.
5. **Project** Find out how drapes are actually sold, and answer Ms. Tall's questions with information from a store in your area.

LESSON 1.9

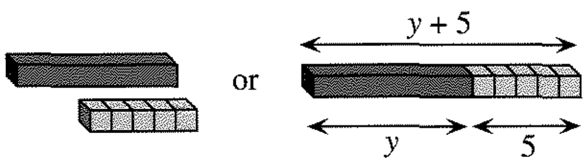
Adding and Multiplying

You will need:

the Lab Gear 

ADDITION

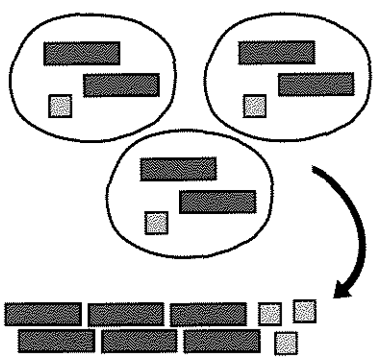
Using the Lab Gear, the addition $y + 5$ can be modeled in two ways. You can show two collections of blocks, y and 5. Or you can line up the blocks to get a figure that has length $y + 5$. Both methods are shown here.



1. Sketch this addition both ways, $3x + 2$.

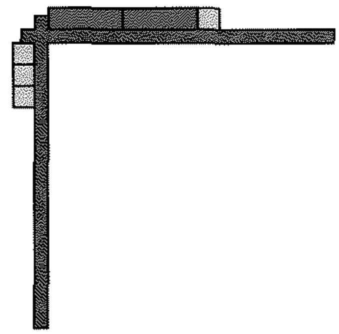
MULTIPLICATION

The multiplication $3 \cdot (2x + 1)$ can be modeled in two ways. One way is to show three collections of $2x + 1$.

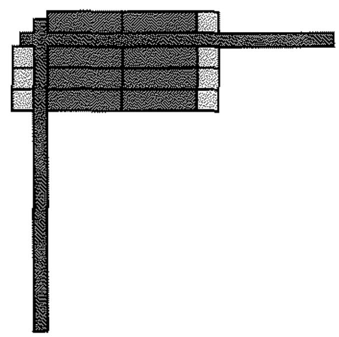


As you can see in the figure, $3 \cdot (2x + 1) = 6x + 3$.

The other way is to use the corner piece. First set up the factors (3 and $2x + 1$) on the outside.



Then make a rectangle having those dimensions.



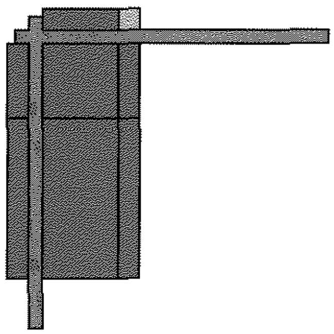
The rectangle represents the product. Again you see that $3 \cdot (2x + 1) = 6x + 3$. This is the familiar *length* $\cdot$ *width* = *area* formula for a rectangle.

2. Sketch this multiplication two ways, $2 \cdot (x + 3)$.
 a. Use collections of blocks.
 b. Use the corner piece.

3. What were the length, width, and area of the rectangle in problem 2?

With any factors of degree 0 or 1, you can model the multiplication in the corner piece.

4. What multiplication is shown in this figure?



5. Multiplying the x by the x gave x^2. What other multiplications do you see in the figure above?

6. Multiply with the corner piece.

a. $3x \cdot 2$ b. $3 \cdot 2x$

c. $2x \cdot 3$ d. $2x \cdot 3y$

7. Multiply with the corner piece.

a. $5(x + 1)$ b. $x(x + 3)$

8. Find the area of a rectangle having the sides given below. For each write an equation of the form *length times width = area.*

a. 5 and $x + 3$

b. x and $2x + 5$

9. Find the sides of a rectangle having the given area. Each problem has at least two solutions. Find as many of them as you can and write an equation for each.

a. $6x$ b. $6x^2 + 3x$

10. These equations are of the form *length times width = area.* Use the Lab Gear to help you fill in the blanks.

a. $y \cdot$ _____ $= y^2 + xy$

b. $(x + 2) \cdot$ _____ $= 3x + 6$

c. (_____ $+ 3) \cdot x = 2xy + 3x$

Understanding the area model of multiplication will help you avoid many common algebra errors.

ORDER OF OPERATIONS

The figure above showed a multiplication. Some students write it like this: $x + 1 \cdot x + y$. Unfortunately, someone else might read it as *add the three terms*: x, $1 \cdot x$, and y. Simplified, this would be $x + x + y$, or $2x + y$. But the intended meaning was equivalent to $x^2 + xy + x + y$, as you can see on the figure. To avoid this kind of confusion, mathematicians have agreed on the following rule.

> **Rule:** When the operations of multiplication and addition (or subtraction) appear in the same expression, *multiplication should be performed first.* If we want to change this order, we have to use parentheses.

This means that one correct way to write the multiplication in the figure is $(x + 1)(x + y)$, which can mean only *multiply* $x + 1$ *by* $x + y$.

11. a. Show $2 \cdot x + 5$ with the Lab Gear. Sketch.

b. Next to your sketch show $2 \cdot (x + 5)$ with the Lab Gear. Sketch it. Keep the blocks on the table for the next problem.

12. a. Copy both collections of blocks from problem 11, substituting 1 for x. What is each expression equal to?

b. Repeat, using 5 for x.

c. Repeat, using 0 for x.

13. Can you find a value of x for which $2 \cdot x + 5 = 2 \cdot (x + 5)$? If so, what is the value? If not, why can't you find a value?

14. **Exploration** Insert parentheses in each expression, so as to get many different values. What are the greatest and smallest values you can find for each one?

a. $0 \cdot 1 + 2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 + 8 \cdot 9$

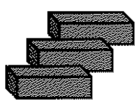
b. $0 + 1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 + 7 \cdot 8 + 9$

THE SAME OR DIFFERENT?

Students sometimes confuse $3 + x$ with $3x$. With the Lab Gear, it is easy to see the difference. $3 + x$ involves addition.

or

$3x$ involves multiplication.



or

15. Find the value of $3 + x$ when:

a. $x = 0$ b. $x = 5$

c. $x = 0.5$

16. Find the value of $3x$ when :

a. $x = 0$ b. $x = 5$

c. $x = 0.5$

17. For most values of x, $3x$ does not equal $3 + x$. In fact there is only one number you can substitute for x that will make $3 + x$ equal to $3x$. Use trial and error to find this number.

18. Build these expressions with the Lab Gear. Sketch. Which two are the same?

a. $6xy$ b. $2x + 3y$

c. $2x \cdot 3y$ d. $5xy$

19. Build and sketch these two expressions with the Lab Gear.

a. $2x + 3y$ b. $2xy + 3$

20. Use trial and error to find a pair of values of x and y that will make the two expressions in problem 19 have the same value.

21. Use the Lab Gear to show each expression. Sketch.

a. $5 + x + y$ b. $5 + xy$

c. $5x + y$ d. $5xy$

22. Choose values for x and y so that all four expressions in problem 21 have different values.

1.10 Three Dimensions

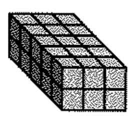
You will need:

the Lab Gear 

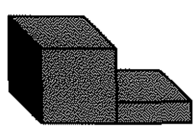
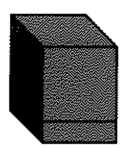
VOLUME

Definition: The *volume* of a solid is the number of unit cubes it would take to build it.

1. What is the volume of this box? Explain how you got your answer.

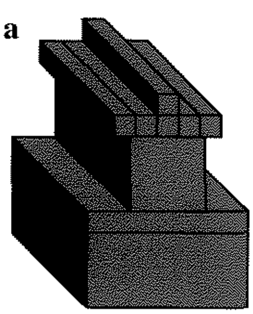


You can find the volume of a Lab Gear building by just adding the volume of each block. For example, both of these buildings have volume $x^3 + x^2$.

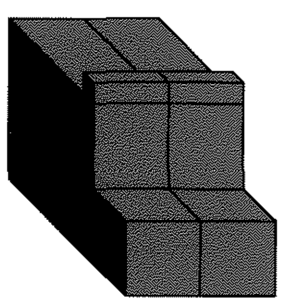
 or 

2. What is the volume of each of these buildings?

a

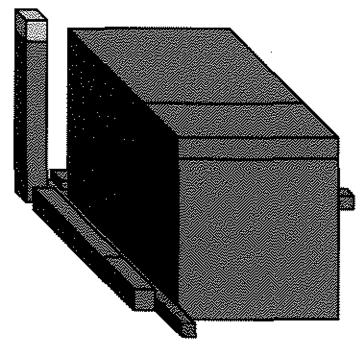


b



MAKE A BOX

Example: This box has volume $y^3 + xy^2 + y^2 + xy$, length $y + x$, width y, and height $y + 1$.



For each problem, the volume of a box is given.

a. Get the blocks.

b. Use them to make a box.

c. Write the length, width, and height.

3. $3xy + x^2y + xy^2$

4. $xy^2 + 2y^2$

5. $x^2y + 2xy + y$

6. $x^2y + xy^2 + xy + y^2$

7. $y^3 + y^2 + xy^2$

8. $x^3 + x^2y + 2x^2 + xy + x$

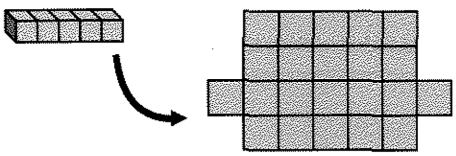
We will return to the volume of boxes in a future chapter.

SURFACE AREA

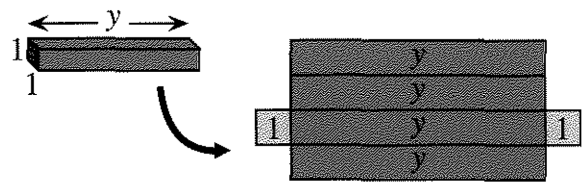
Definition: The *surface area* of a solid is the number of unit squares it would take to cover all its faces (including the bottom).

In simple cases, to figure out the surface area it helps to think of a paper jacket that would cover the whole block. The area of such a jacket is the surface area of the block.

For example, the surface area of the 5-block is 22 cm^2. Its volume, of course, is 5 cm^3.



9. Find the surface area of the 25-block.



The surface area of the blue blocks can also be figured out by thinking of their jackets. For example, the *y*-block has a surface area of $4y + 2$.

10. Find the surface area of each of the other blue Lab Gear blocks.

DISCOVERY *POLYCUBES*

Definition: *Polycubes* are obtained by joining cubes together face-to-face. They are the three-dimensional equivalent of polyominoes. Here is a *tetracube.*

There is just one *monocube,* and one *dicube.* There are two *tricubes* and eight *tetracubes.*

All of these polycubes look just like the corresponding polyominoes, except three of the tetracubes, which are really three-dimensional.

11. Find all the polycubes, monocube to eight tetracubes, with your blocks and try to sketch them. Hint: Two of the three-dimensional tetracubes are mirror images of each other.

12. Find the surface area of the polycubes you found in problem 11.

13. Find polycubes having volume 8 and as many different surface areas as possible. There are five different solutions.

14. Were any of your surface areas odd numbers? If yes, check your work. If no, explain why not.

15. For a given number of cubes, how would you assemble them to get the largest surface area? The smallest?

16. What would the largest possible surface area be for a polycube having volume 100?

17. Explain in words how you would find the largest possible surface area for a given volume.

18. For each of the following volumes, find the smallest possible surface area.

a. 12 b. 18 c. 20
d. 24 e. 27 f. 30

19. Explain in words how you would find the smallest possible surface area for a given volume.

MORE ON POLYCUBES

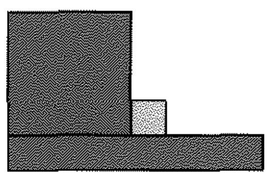
20. Find all the polycubes having volume less than 5. Put aside all the ones that are box-shaped. The remaining pieces should have a total volume of 27. Using wooden cubes and glue, make a set of puzzle pieces out of these polycubes. Assemble them into a 3-by-3-by-3 cube. (This classic puzzle is called the Soma® Cube.)

21. There are 29 pentacubes. Twelve look like the pentominoes, and 17 are "truly" three-dimensional. Find them all and sketch them.

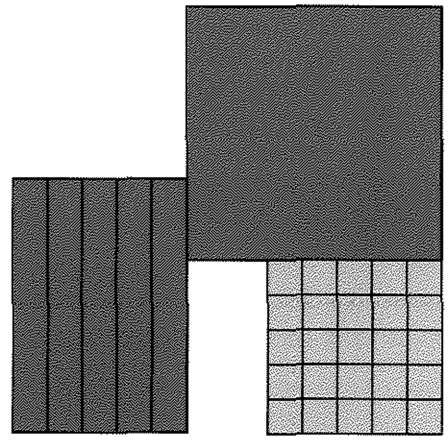
REVIEW *PERIMETER*

Find the perimeter of each figure.

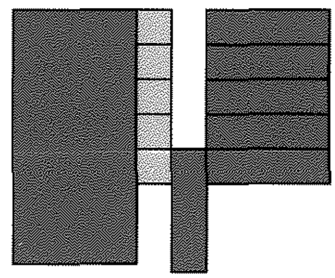
22.



23.



24.



LESSON 1.11

Word Figures

WORD TRIANGLES

Imagine you have a supply of letter tiles, and you use them to make word triangles like this one.

A
L A
A L E
R E A L

Rules: Each row contains the letters of the previous one, plus one more. It's OK to scramble the letters from one row to the next.

1. Extend this word triangle.
2. Make a word triangle with your own letters.
3. How many letter tiles are used in a five-row word triangle?
4. Make a table like this, extending it to ten rows.

Rows	Tiles
1	1
2	3
3	6
4	...

5. The numbers you found in problem 4 (1, 3, 6, ...) are called the *triangular numbers*. Explain how they are calculated.
6. Extend the above word triangle up to **ARGUABLE**. (Along the way, you might use **ALGEBRA**.)

WORD LADDERS

Rules: From one row to the next, change one letter only. It's OK to scramble the letters.

For example:

R E A L
L E A D
L O A D
F O L D
F O O D

7. Make up a word ladder with your own letters. Choose your word length, and use as many rows as you need. It's fun to choose related words for the beginning and end of the ladder, like **CAR** and **BUS**.

The above example, from **REAL** to **FOOD**, took four steps (and five rows). It is an example of a *perfect* word ladder. For a word ladder to be called perfect, two things must be true:

a. Every letter from the original word must be changed in the final word.
b. If the word has n letters, the ladder must take exactly n steps.

For a five-letter word, a perfect ladder would take five steps (one per letter) and therefore six rows.

8. How many tiles would a five-letter perfect word ladder require?
9. Make a table of the number of tiles required for perfect word ladders, extended to word length 10.

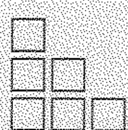
Word Length	Tiles
1	2
2	6
3	12
4	...

10. The numbers you found in problem 9 (1, 6, 12, ...) are called the *rectangular numbers*. Explain how they can be calculated.

11. Make up a word ladder from **MATH** to **GAME**.

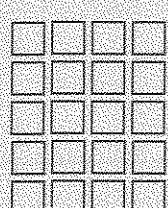
PUTTING IT TOGETHER

12. This figure shows the third triangular number.



Draw a sketch of two copies of this triangle, arranged together to make a rectangle.

13. This figure shows the fourth rectangular number.



Show how you could divide it into two equal triangular numbers.

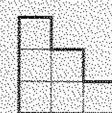
14. **Summary** Describe the relationship between triangular numbers and rectangular numbers.

15.
a. Explain how to calculate triangular numbers by first calculating rectangular numbers.
b. Calculate the 100th triangular number.

POLYOMINO PUZZLES

16. Using graph paper and scissors, or interlocking cubes, make a set of polyominoes having area greater than 1 and less than 5. You should have one domino, two trominoes, and five tetrominoes, for a total of eight puzzle pieces with no duplicates.

17. Using the same unit as you used for the puzzle pieces, draw staircases with base 3, 4, 5, 6, and 7. The first one is shown here.



Now cover each staircase in turn with some of your puzzle pieces. Record your solutions on graph paper. For the last staircase, you will need all of your pieces.

18. Make a list of all the rectangles, including squares, having area 28 or less. Their dimensions (length and width) should be *whole numbers greater than 1.* (In other words, the shortest sides should be 2.) There are 25 such rectangles.

19. Draw these rectangles, and use the puzzle pieces to cover them. Record your solutions on graph paper. (It is impossible to cover one of the rectangles.)

POLYOMINO AREA AND PERIMETER

Think of the monomino. Its area is 1 and its perimeter is 4. Think of the domino. Its area is 2 and its perimeter is 6.

20. **Exploration** Is the number representing the perimeter of a given polyomino always greater than the number representing its area, or can it be equal to it, or even smaller? Look over your notes and sketches from Lesson 2, and experiment some more on graph paper if you need to. Then write a paragraph to answer this question fully, with examples and graph paper illustrations.

21. Find out if there are polyominoes having both area and perimeter equal to

a. 14 b. 15 c. 16

d. 17 e. 18 f. 20

WORD SQUARES

This is a word square.

	a	b	c	d
a	M	A	T	H
b	A	C	R	E
c	T	R	E	E
d	H	E	E	L

Note that the words can be read across or down. The largest word square in the English language took years of hard work to discover. It is made up of obscure ten-letter words.

22. How many letter tiles are used in the word square above?

23. Make a table showing the number of tiles required for word squares, extended to word length 10.

Word Length	Tiles
1	1
2	4
3	9
4	...

24. The numbers you found in problem 11 (1, 4, 9, ...) are called the *square numbers.* Explain how they can be calculated.

25. There is an interesting pattern based on adding pairs of consecutive triangular numbers (1 + 3, 3 + 6, ...) Explain it.

26. Draw a sketch of the third triangular number put together with the fourth triangular number (upside down) to show a square number.

27. What do you think the 100^{th} triangular number and the 101^{st} triangular number add up to?

28. Make a word square using these clues. The answer words are all four letters long and can be read both across and down.

a. Made to be played.

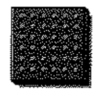
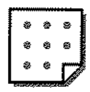
b. You learned about it in this chapter.

c. Don't make one!

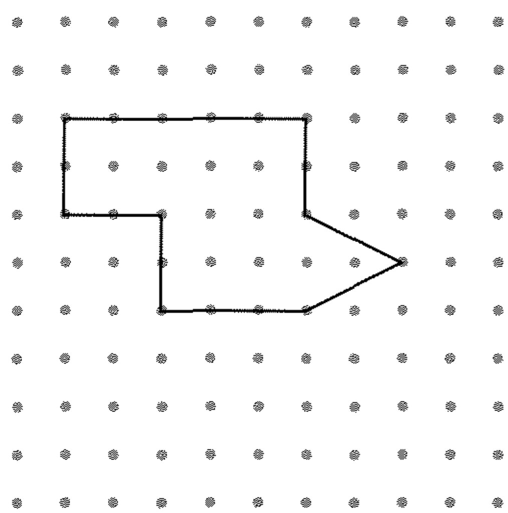
d. A piece of cake.

1.12 Area on the Geoboard

You will need:

- geoboard 
- dot paper 

This geoboard shape has area 18.



1. **Exploration** Find as many geoboard shapes having area 18 as you can. They do not need to be rectangles. You are allowed to stretch the rubber band in any direction whatsoever, including diagonals. Sketch each shape on dot paper.

TRIANGLES

2. On your geoboard make three triangles, each one satisfying one of the following conditions. Sketch each triangle on dot paper.
   a. One side is horizontal, and one is vertical.
   b. One side is horizontal, no side is vertical.
   c. No side is horizontal or vertical.

3. Repeat problem 2 for these conditions.
   a. Two sides are of equal length, one horizontal and the other vertical.
   b. Two sides are of equal length, but neither is horizontal or vertical.

VERTICES

Definition: The corners of geometric figures such as triangles and rectangles are called *vertices.* (Singular: *vertex.*)

4. Make a figure on the geoboard having vertices in order at (4, 6), (7, 5), (8, 3), (8, 2), (6, 0), (2, 0), (0, 2), (0, 3), (1, 5).

5. Do not remove the rubber band from problem 4. Using another rubber band, make a figure having vertices in order at (2, 2), (6, 2), (5, 1), (3, 1).

6. Add eyes to the face. What are the coordinates of their vertices?

AREA TECHNIQUES

7. Make a triangle having vertices at (0, 0), (0, 10), and (10, 0). What is its area? Explain how you figured it out.

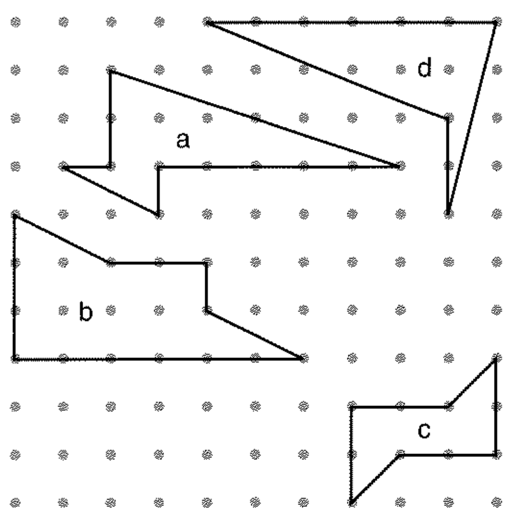
8. Make a triangle having vertices at (0, 10), (0, 6), and (3, 6).
   a. With another rubber band, make a rectangle that shares three of its vertices with the triangle. What are the coordinates of the fourth vertex of the rectangle?
   b. What is the area of the rectangle?
   c. What is the area of the triangle?

9. Find the area of a triangle having vertices at (0, 10), (0, 5), and (7, 5).

10. On your geoboard, make two different-shaped triangles that satisfy these conditions: one horizontal and one vertical side, and area 10. Record your solutions on dot paper.

11. Repeat problem 10 for area 9.

12. Copy these figures on your geoboard (or on dot paper). Find the area of each one. Explain how you did it.



13. On your geoboard, make the triangle having vertices at (0, 10), (0, 4), and (3, 6).

a. With another rubber band, divide the triangle into two smaller triangles, such that they each have one horizontal and one vertical side. Find the area of all three triangles.

b. With another rubber band, make the smallest rectangle that covers the original triangle. What is the area of the rectangle?

14. Find the area of the triangle having vertices at (0, 0), (0, 7), and (3, 5).

15. Record your solutions on dot paper.

a. Make five triangles having a horizontal side of length 6 and area 15.

b. Make five triangles having a horizontal side of length other than 6 and area 15.

c. Make five triangles having a vertical side of length 7 and area 10.5.

16. Find the area of the triangle having vertices at (0, 0), (0, 5), and (3, 7).

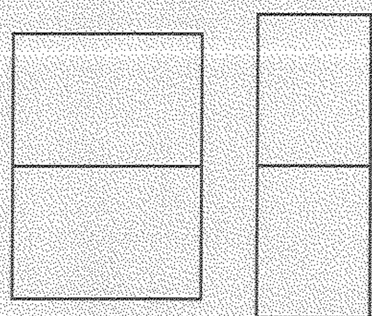
17. **Summary** Explain how one finds the area of a geoboard triangle having one horizontal or vertical side.

THINKING WRITING

1.C More Window Prices

In Lesson 8 you figured out how window prices were determined in an imaginary store. Real prices are probably not determined this way.

Window manufacturers use a special four-number code for describing the size of standard two-pane windows like those shown below. The first two numbers give the width in feet and inches, and the last two numbers give the height. For example, the code 2636 means that the window is 2 feet 6 inches wide and 3 feet 6 inches high.



The prices for some windows are given below. You will investigate how the price depends on the dimensions of the window.

Code	Price
3030	$108.00
4030	$135.00
3040	$130.50
4040	$162.00

1. What are the dimensions of a 1640 window?

2. Use the code to figure out the dimensions of the windows. Make a table showing the code, the dimensions, and the price. You may also want to include other measurements, like the perimeter or area.

3. Experiment to figure out how the prices were determined. (The formula is not the same as the one used by the A.B. GLARE window store.) Try to find a pattern. According to your pattern, what should a 3050 window cost?

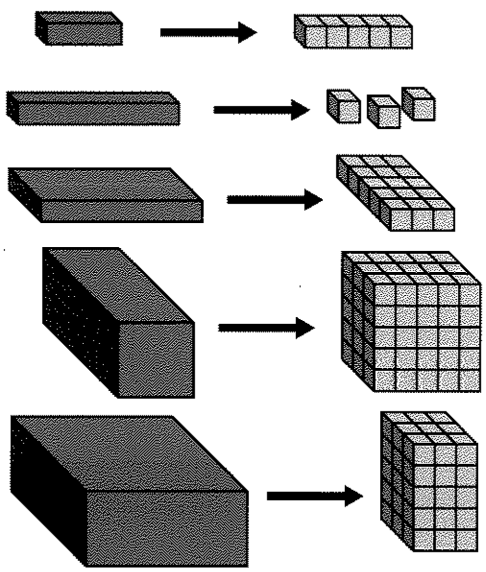
4. **Report** Write a report about this problem.
   - First, clearly state the problem you are solving.
   - Next, explain the results of your investigation. Include the table you made and explain how you used it to find a formula relating the code to the price. Include sketches and show your calculations in a systematic way. Give a couple of examples to illustrate that your formula really works. Explain why the order of the numbers in the code is important. For example, compare the cost of a 3050 window with the cost of a 5030 window. Make another price list showing what some other windows should cost.
   - Write a brief conclusion commenting on your results. Explain why this method of pricing makes sense. Would it still make sense for very large or very small windows? If you do not think so, can you think of a better way?

Essential Ideas

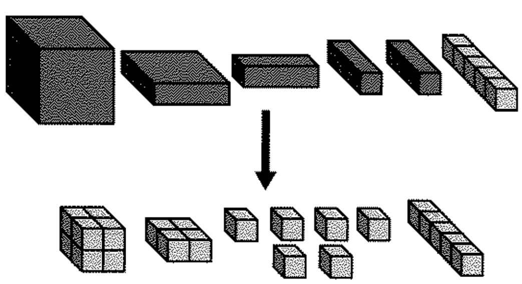
VARIABLES AND CONSTANTS

1. Explain, with examples, how to figure out the names of the other blue blocks by using 5-blocks, x-blocks, y-blocks, and the corner piece.

If $x = 5$ and $y = 3$, we can use the Lab Gear to think of xy, x^2y, xy^2.



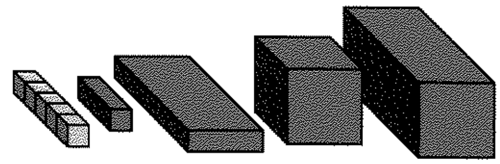
Here is $x^3 + x^2 + 3x + 5$, if $x = 2$.



For each problem, write what the blocks show in terms of the variables x and y, then use substitution to evaluate them for:

a. $x = 0$ and $y = 2$;
b. $x = 5$ and $y = 1$;
c. $x = 2$ and $y = 3$.

2.



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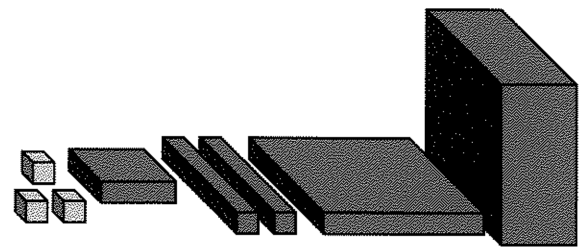
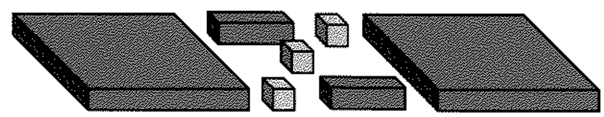


LIKE TERMS

Combine like terms.

4.



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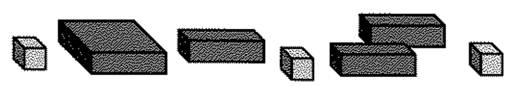

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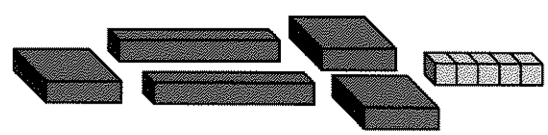

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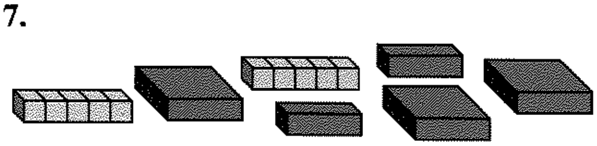

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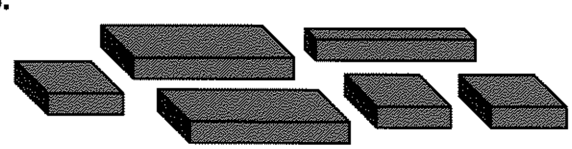

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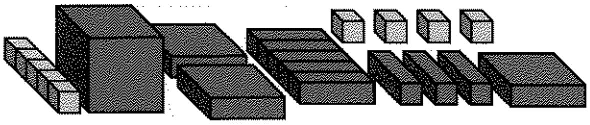

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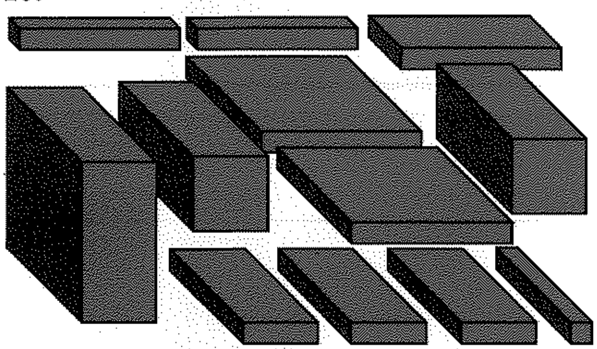

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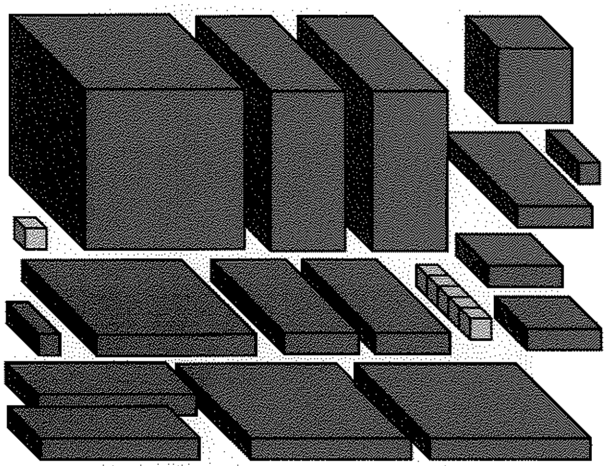


UNLIKE TERMS

12. My student Al *doesn't* like terms. He missed every problem on the Algebra Quiz. Please help poor Al with his algebra. For each problem, give the correct answer and explain what Al did wrong. Use Lab Gear sketches when possible.
   a. $x^2 + x = x^3$
   b. $3x + x = 3x^2$
   c. $x^2 + x^2 + x^2 + x^2 = x^8$
   d. $y \cdot 6 = y^6$
   e. $2x + 3y = 5xy$

ADDING AND MULTIPLYING

13. Use the Lab Gear to show each expression. Sketch.
   a. $2 + x + y$
   b. $2 + xy$
   c. $2x + y$
   d. $2xy$

14. a. Find values for x and y so that all four expressions in problem 13 have different values.
   b. Find values for x and y so that as many as possible of the given expressions are equal to each other.

15. Use the Lab Gear to show each expression. Sketch. (Hint: Use the corner piece for the last one.)
   a. $x + y^2$
   b. $x^2 + y$
   c. $x^2 + y^2$
   d. $(x + y)^2$

16. Find values for x and y so that all four expressions in problem 15 have different values.

ORDER OF OPERATIONS

17. Use the Lab Gear to show $2 + 5y$ and $(2 + 5)y$. Sketch each one.

18. a. Use trial and error to find a value of y such that $2 + 5y = (2 + 5)y$.
   b. If $y = 0$, which is greater, $2 + 5y$ or $(2 + 5)y$?
   c. If $y = 2$, which is greater, $2 + 5y$ or $(2 + 5)y$?

AREA AND MULTIPLICATION

Use the corner piece for problems 19-21.

19. Find the area of a rectangle having the sides given below. For each problem write a multiplication of the form *length times width = area.*

a. 3 and 5 b. 3 and x

c. 3 and $x + 5$ d. x and $x + 5$

20. Find the sides of a rectangle having the following areas. Each problem has at least two solutions. Find as many of them as you can and write an equation for each.

a. $4x$ b. $4x^2 + 8x$

c. $3xy + 6x^2 + 9x$

21. These equations are of the form *length times width = area.* Use the blocks to help you fill in the blanks.

a. $x \cdot$ _____ $= x^2 + xy$

b. $(y + 1) \cdot$ _____ $= 5y + 5$

c. (_____ $+ 3) \cdot y = 2xy + 3y$

d. $2x \cdot$ _____ $= 4x + 2xy + 6x^2$

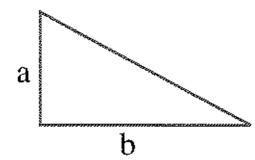
22. Use the Lab Gear to build all the rectangles (or squares) you can find having the following perimeters. For each one, sketch your answer and write the length, width, and area.

a. $8x$

b. $6x + 2y$

c. $4x + 4y$

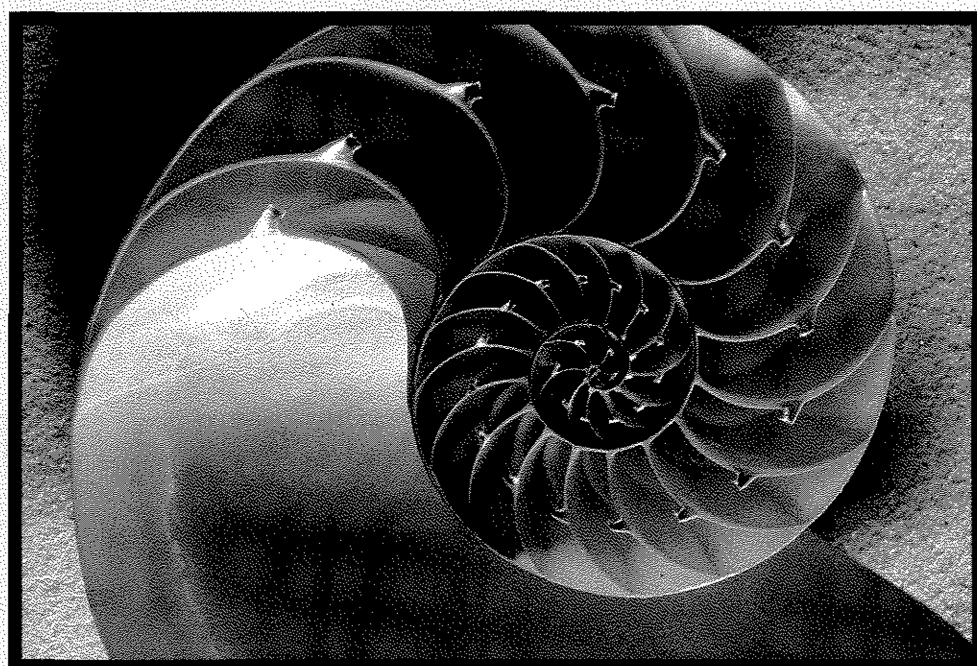
23. What is the area of the triangle in the figure if



a. a = 7 and b = 9?

b. a = $4x$ and b = y?

CHAPTER 2



The equiangular spiral of a nautilus shell

Coming in this chapter:

Exploration 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377... There are many patterns in this sequence of numbers. Find as many as you can, using addition, subtraction, multiplication, and division.