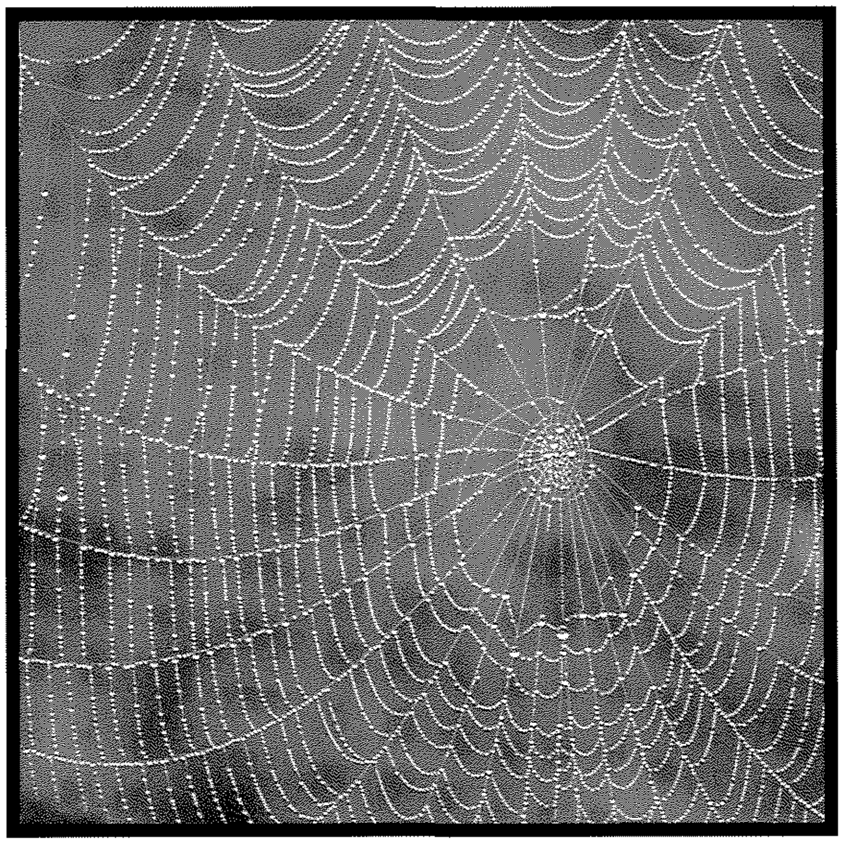
CHAPTER 6



The outward spiral path of a spider web

Coming in this chapter:

Exploration A movie discount card, valid for three months, costs $\$T$. With the card, it costs only $\$D$ to attend a movie, instead of \$5. How many movies would you have to see in three months in order to save money with the discount card? Does your answer depend on T, D, or on both?

MAKING COMPARISONS

LESSON 6.1

Comparing Car Rentals

You will need:

graph paper

This table gives the results of a phone survey of the cost of renting a mid-size car in a large city.

Company	Daily rate	"Free" miles	Cost per additional mile
A	$34.99	150 miles	24 cents
B	$26.95	100 miles	30 cents
C	$39.95	100 miles	30 cents
D	$41.95	unlimited mileage	—
E	$27.99	unlimited mileage	—

1. **Exploration** Suppose you wanted to rent a car for a short trip and you had the information in the table. There is one car that is clearly the "best deal" in most cases. Which car is this? If this car were not available, how would you decide which car to rent? Write a paragraph explaining how you would decide. Include the following:
   - What things would you consider?
   - Show any calculations you would need to do to make your decision.
   - Is there any additional information not included in the table that you think you would need to know?

USING TABLES

2. Which car do you think would be the best deal if you planned to drive a short distance? Which car would you rent to drive several hundred miles? Explain.

This table gives the cost of renting each car for one day to drive the indicated number of miles.

3. Copy and complete the table, indicating how much it would cost to rent each car for the given miles.

Total Cost of Car Rental

Company	Miles driven				
	50	100	150	200	250
A	34.99	34.99	34.99	46.99	58.99
B	26.95	26.95	41.95	56.95	—
C	—	—	—	—	—
D	—	—	—	—	—
E	—	—	—	—	—

4. Copy and complete the next table. It ranks each car according to the amount it would cost to rent it to drive the given number of miles. The code is **1** for least expensive and **5** for most expensive.

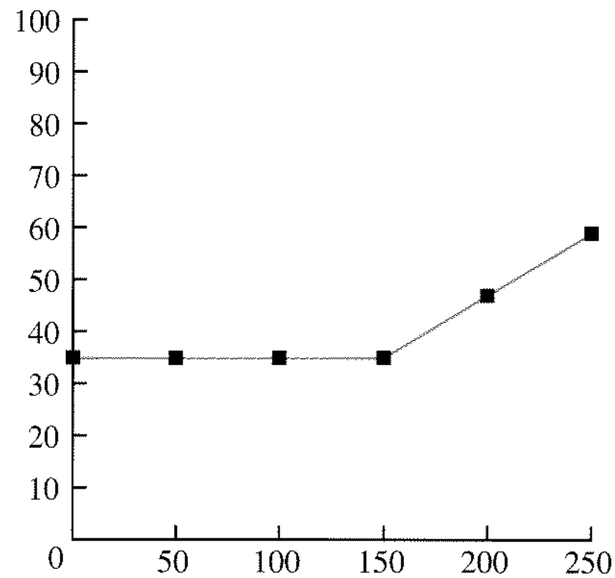
Company Rankings

Company	Miles driven				
	50	100	150	200	250
A	3	—	—	—	—
B	1	—	—	—	—
C	4	—	—	—	—
D	5	—	—	—	—
E	2	—	—	—	—

5. In the table you just completed, you can see that B is less expensive than A for 50 miles and 100 miles of travel, but this is reversed for 150 miles of travel.
   a. Which is less expensive for 125 miles of travel?
   b. Show that the costs of A and B are almost exactly the same for 130 miles of travel.

USING GRAPHS

6. The graph above problem 7 shows, for a single day of rental, how the cost of renting a car from Company A varies as a function of the number of miles driven.
   a. Make an enlarged copy of the graph on your own paper.
   b. Add to the same grid a similar graph for each of the other four companies. Your graphs must be accurate.



7. Two of the graphs should be horizontal lines. Which ones, and why?

8. According to your graphs, if you plan to drive 100 miles or less,
   a. which company is the most expensive?
   b. which company is the least expensive?

9. Company A has a higher daily rate and lower mileage costs than Company B.
   a. Which of the two is more expensive for someone who travels 100 miles?
   b. Which is more expensive for someone who travels 150 miles?
   c. For what length trip is the cost of the two the same?

10. Company D has a slightly higher daily rate than Company C, but its mileage costs are zero.
    a. For what length trip is D cheaper?
    b. For what length trip are they the same?

11. The graphs for B and D intersect at the point (150, 41.95).
   a. Label this point of intersection on your graph.
   b. Label other points of intersection on any of the other graphs.
   c. How would you interpret these points of intersection in terms of cost comparisons?

12. In what ways are tables better than graphs in helping you make a decision of this type? In what ways are graphs better?

USING EQUATIONS

Equations are useful if you want to use a computer or a programmable calculator to help you analyze a problem like this one. You can write an equation for the cost of renting a car from Company A for one day as a function of the number of miles traveled. Notice that the graph has two parts: a horizontal part, and a part that slopes upward. The equation also has two parts.

If y is the cost in dollars and x is the number of miles driven, then:

$$y = 34.99 \qquad \text{if } x \leq 150$$
$$y = 34.99 + 0.24(x - 150) \qquad \text{if } x > 150$$

13. Which part of the equation represents the horizontal part of the graph?

14. Explain every part of the second equation. (Why is 150 subtracted from x? Why are parentheses necessary? What is the meaning of the quantity in the parentheses? Why is it multiplied by 0.24? Why is the result added to 34.99? What is the meaning of the sum?)

15. Write equations for the costs of renting the other cars as a function of miles driven.

DISCOVERY *GRADE AVERAGES*

Mrs. Washman gives a quiz every Thursday. A student's current average at the end of any week can be computed by finding the ratio of *total correct points* to *total possible points* to date. The table shows Caden's scores.

	Q1	Q2	Q3	Q4	Q5
Correct	8	9	10	13	—
Possible	12	20	10	15	—

16. Find Caden's current average at the end of week 1, week 2, week 3, and week 4.

17. Caden found his current average by doing this computation:

$$\frac{8}{12} + \frac{9}{20} + \frac{10}{10} + \frac{13}{15} = \frac{40}{57}$$

Amiko said this was wrong because you don't add fractions by adding the numerators and adding the denominators. Who was right? Were they both right? Were they both wrong? Explain.

18. What would Caden's average be at the end of week 5 if Quiz 5 had
   a. 20 points, and he got 12 correct?
   b. 40 points, and he got 80% correct?
   c. 25 points, and he got N correct?

LESSON 6.2

Which is Greater?

You will need:

the Lab Gear

Exploration For each problem, if possible, give one value of x that

a. makes the right side greater;
b. makes the left side greater;
c. makes the two sides equal.

Describe the method you used for each problem.

1. x **?** $2x + 3$
2 $y - 2$ **?** $-y - 2$
3. $6x$ **?** $7x^2 + 6x - 7$

USING THE LAB GEAR

For each problem:

a. Simplify each expression.
b. Compare the two expressions. It may help to build them with the Lab Gear, one on each side of the workmat.
c. Is one side greater, or are they equal? Write the correct symbol: >, <, or =. If it is impossible to tell, write **?**. Remember that x is not necessarily a positive integer.

4. $x(x + 2) - 4$ $\quad$ $(x + 1)(x + 1)$
5. $(x + 1)(x + 2)$ $\quad$ $2 + 3x - x^2$
6. $3x^2 + 9 - (x^2 + 2)$ $\quad$ $3x^2 + 9 - x^2 + 2$
7. $3x^2 + 9 - (x^2 + 2x)$ $\quad$ $3x^2 + 9 - x^2 + 2x$
8. If you did not get at least one **?** as an answer in problems 4-7, check your work.

USING TABLES

For which values of x is $14x - [4x - (2 - 3x)]$ greater than $5x - 2[x - (3x + 2)]$?

These two expressions are too complicated to build with the Lab Gear. It is easier to compare them if they are simplified first. Both expressions have two sets of grouping symbols, *parentheses* and *brackets*. Brackets mean exactly the same thing as parentheses.

Rule: Simplify from the inside out, removing the parentheses first.

9. Removing parentheses, the first expression is $14x - [4x - 2 + 3x]$. Continue simplifying.
10. Removing parentheses, the second expression is $5x - 2[x - 3x - 2]$. Continue simplifying.

The table below compares the expressions $9x + 4$ and $7x + 2$ for some values of x.

x	$9x + 4$	$7x + 2$
10	94	72
5	49	37
0.1	4.9	2.7

11. Copy and extend the table.
    a. Find some values of x for which $9x + 4$ is less than $7x + 2$.
    b. Try to find a value of x for which the two expressions are equal.
    c. Describe any patterns you see in your table.

Lea and Earl were trying to compare these expressions:

Expression A: $5 - [x - (3x + 1)]$

Expression B: $5 - 3[x - (3x + 1)]$

They got different results when they simplified Expression B.

Simplifying Expression B

	Lea's work	Earl's work
Step 1	$5 - 3[x - 3x - 1]$	$5 - 3[x - 3x - 1]$
Step 2	$5 - 3[-2x - 1]$	$5 - 3[-2x - 1]$
Step 3	$5 + 6x - 3$	$2[-2x - 1]$
Step 4	$2 + 6x$	$-4x - 2$

Lea and Earl wanted to know which one of them had made a mistake. They asked their teacher, Mr. Martin. "You can't both be right," he said, "but you could both be wrong."

12. Are Lea and Earl both wrong, or is only one of them wrong? Is Mr. Martin wrong? Look for mistakes in their work. When you find a mistake, explain what the student did wrong.

13. Look at Expressions A and B again. Simplify both expressions correctly.

14. Using the simplified form of each expression, compare Expressions A and B by making a table of values.

15. Summarize the information in your table by telling when Expression A is greater, when Expression B is greater, and when the two expressions are equal.

16. Simplify each pair of expressions.

a. $4x - 2x[3 - 6(x + 1)]$
   $4 - x[x - 6(2x + 1)]$

b. $4 - 2[y - 6(y + 1)]$
   $4 - [y - 6(y + 1)]$

17. Compare each pair of expressions in problem 16. Make a table of values and summarize your findings in each case, telling when the first expression is greater, when the second expression is greater, and when they are equal.

SOLVING INEQUALITIES

18. Use a table of values to show that $2x + 6 > 8$ for all values of x greater than 1.

We say that the *solution* to the inequality $2x + 6 > 8$ is "all numbers greater than 1" because this describes *all* the values for which the inequality is true. Using mathematical symbols, we say that the solution is $x > 1$.

Find the solution of each inequality. That is, describe all the numbers for which the inequality is true.

19. $x + 5 > 1$ **20.** $n - 5 > 1$

21. $y + 5 > 0$ **22.** $r - 5 > 0$

23. $x - 5 > -1$ **24.** $x + 5 > -1$

25. $-x > 6$ **26.** $-x > -6$

27. Many students get problems 25 and 26 wrong. Check your answers to them by substituting specific values of x. What makes them more difficult than the other ones?

DISCOVERY *SQUARES ON A CHESSBOARD*

28. How many squares of any size are there on an 8-by-8 chessboard? Explain how you get your answer. (Hint: First analyze smaller boards.)

LESSON 6.3

Solving Linear Equations

You will need:

the Lab Gear 

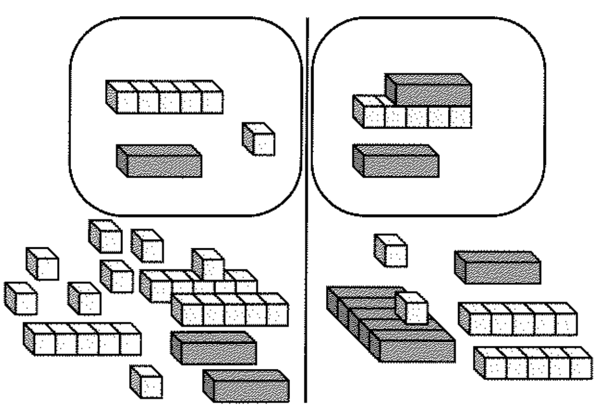
1. **Exploration** Find a value of x that makes each equation true. Describe the method you used.
   a. $3(x + 2) = x + 5$
   b. $3x + 2 = x + 5$
   c. $3x + 2 = x - 5$
   d. $3(x + 2) = x - 5$

USING THE LAB GEAR

The easiest equations to solve are linear, or first-degree equations in one variable. All four of the equations above are linear. The equation $x^2 = 2x - 1$ is not linear, because it contains an x^2 term.

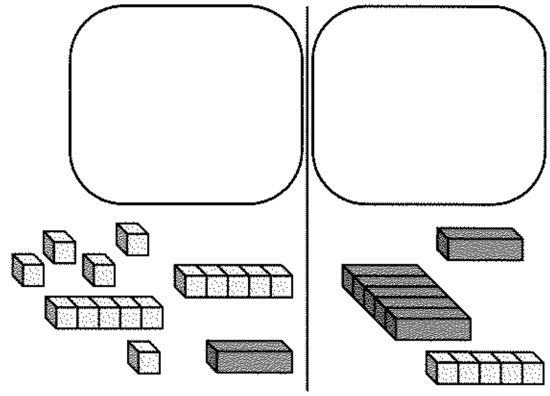
You have already learned to solve equations by trial and error and the cover-up method. Some kinds of equations can also be solved using the Lab Gear.

This figure represents an equation. We want to find out what value of x will make the quantity on the left side of the workmat equal to the quantity on the right side.

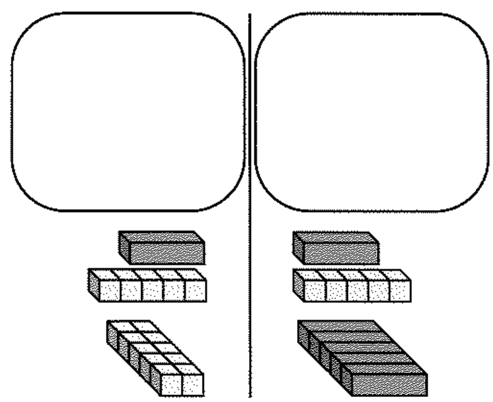


2. Copy the figure with your Lab Gear.

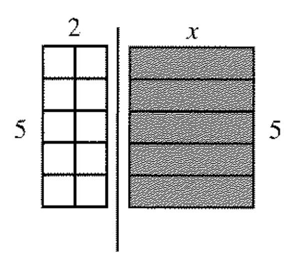
3. Simplify each side. If you did it correctly, your blocks should match this figure.



4. Rearrange the blocks to match this figure. Which blocks on the right side can be matched with identical blocks on the left side?



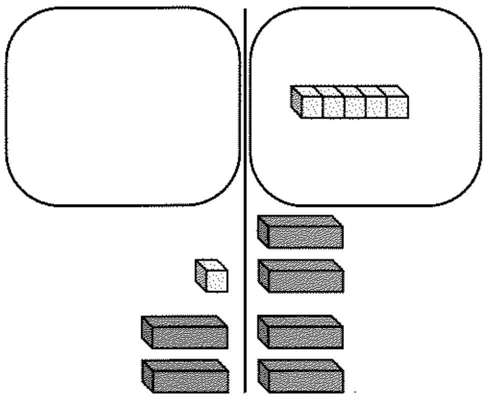
There are some blocks that cannot be matched with blocks on the other side. The figure shows a two-dimensional view of these blocks.



Look at these remaining blocks. Remember that the two sides are equal. This is true even though they don't *look* equal. Remember x can have any value.

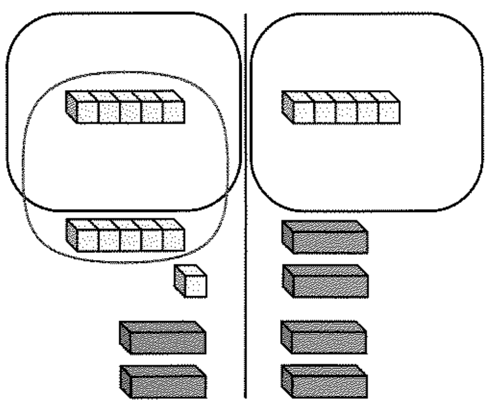
5. What must x be in order for the two sides to be equal?

This figure shows how you would set out the blocks to solve the equation $2x + 1 = 4x - 5$.



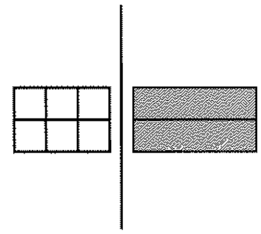
Each side is simplified. The blocks have been arranged to show which blocks can be matched with blocks on the other side. Even so, it is not easy to tell what the solution is.

It helps to add zero.



Notice that the blocks on one side are rearranged to show which ones can be matched with blocks on the other side.

The remaining blocks (those that cannot be matched with blocks on the other side) can then be rearranged to make it easy to see the solution to the equation.



6. What is the solution to this equation?

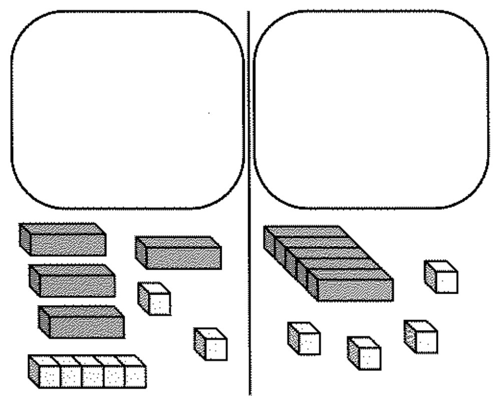
For problems 7-11:

a. Write the original equation.

b. Use the Lab Gear to find the solution. Write equations to show some of the steps as you move your blocks.

c. Write the solution.

7.



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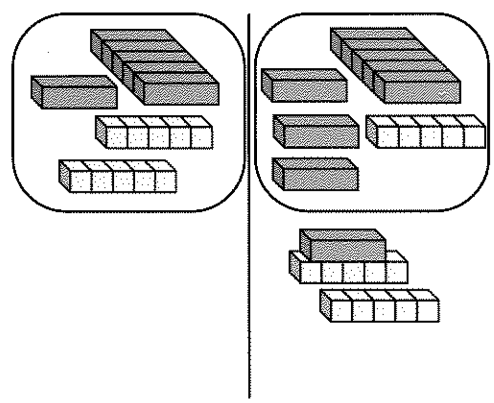

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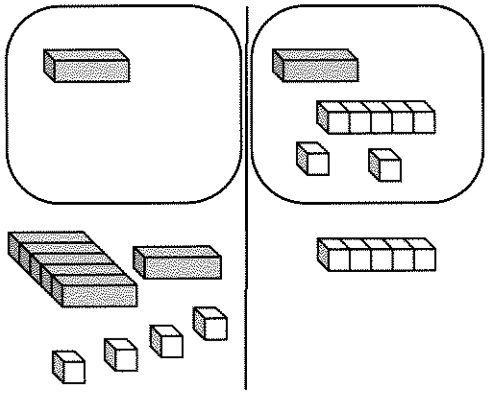

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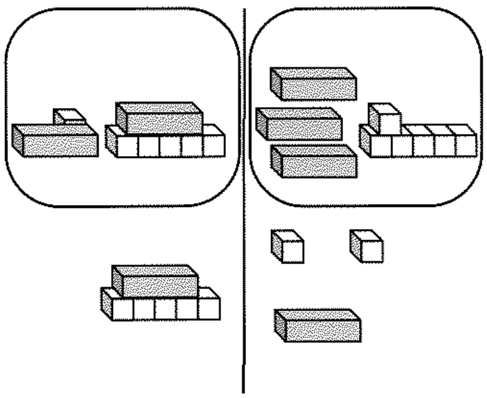

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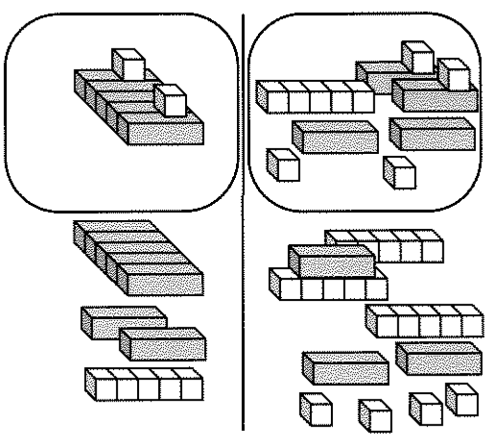


MORE EQUATIONS

You have learned to solve equations using tables, trial and error, the cover-up method, and the Lab Gear. Solve these equations in whatever way you want, but *show your work.*

12. $3x + 5 = 6$

13. $3x + 5 = -2x - 10$

14. $2y - 6 = 5y + 3$

15. $\frac{6x - 6}{4} = 3$

16. $\frac{6x - 6}{4} + 15 = 3$

Make up an equation satisfying each of the following descriptions. Try to make up one that would be challenging for another student to solve, but not so challenging that you can't solve it.

17. An equation whose solution is $x = 4$

18. An equation whose solution is $y = -1/2$

19. An equation that has variables on both sides of the equation and the solution $m = 2$

20. An equation that has more than one solution

DISCOVERY USING VARIABLES

21. A student is x years old. How many months old is he? (Are you sure?)

22. Another student is y years old.
   a. How many years until she can vote?
   b. How many months?

23. I start with 99 peanuts. It takes me x seconds to eat one.
   a. How long will it take to eat them all?
   b. After eating n peanuts, how many are left?
   c. After z seconds, how many peanuts are left?

PUZZLES MAGIC SQUARES

These puzzles will be easier to solve if you make yourself little squares of paper with numbers written on them. To solve the puzzles, move the papers around, until you find a satisfactory arrangement.

24. Arrange all the numbers from 1 to 9 into a 3-by-3 square, so that the sum of all the numbers in any row or column is always the same.

25. Repeat problem 24, but make sure the *diagonals* also add up to the same amount.

26. Arrange all the numbers from 1 to 16 into a 4-by-4 square, so that the sum of the numbers in any row or column is always the same.

DISCOVERY GRADING POLICIES

At the Shell School, math teachers give a six weeks grade based on six quizzes and two writing assignments. The math department policy requires that quizzes and writing assignments be counted equally.

For each student in her class, Mrs. Washman averages the quizzes, averages the writing assignments, and then adds those two numbers and divides by two.

For each student in his class, Mr. Pitcher adds all the grades together and divides by eight.

27. Make up a list of grades of a student who would have a higher grade with Mrs. Washman's method.

28. Make up a list of grades of a student who would have a higher grade with Mr. Pitcher's method.

29. Is it possible for a student to have the same grade using either method? Explain.

LESSON 6.4

Equations and Identities

You will need:

the Lab Gear 

Definition: An *identity* is an equation that is true for all values of the variables.

1. Which of these equations are identities? Explain your answers.

a. $3x + 9 - 2(x + 2) = 3x + 9 - 2x + 2$

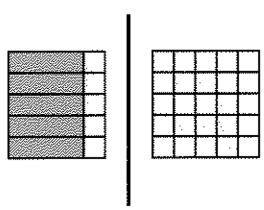
b. $3x + 9 - 2(x + 2) = 3x + 7(x + 2)$

c. $3x + 9 - 2(x + 2) = x + 5$

d. $3x + 9 - 2(x + 2) = x + 7$

USING THE LAB GEAR

To solve the equation $5(x + 1) = 25$ you can model both the left side and the right side as rectangles. In this case, you can match the rectangles, and it is easy to see what the value of x must be.



2. What is the value of x that makes both sides equal?

Use the Lab Gear to solve these equations. If the equation is an identity, explain how you know, using sketches if necessary.

3. $3(x + 2) = 15$

4. $3(x + 2) = 3x + 6$

5. $4(2x + 1) = 4(x + 5)$

6. $4(2x - 1) = 4(x - 1)$

7. $4(2x - 1) = 4(2x + 1)$

8. $2(2x + 2) = 4(x + 1)$

9. $4(2x - 2) = 2(4x - 4)$

USING GRAPHS AND TABLES

10. Make a table of (x, y) pairs and graph each linear function.

a. $y = -2(x - 1) + 2$

b. $y = -2x + 4$

11. By simplifying the left side, show that $-2(x - 1) + 2 = -2x + 4$ is an identity.

12. For each pair of functions, decide whether or not both members of the pair would have the same graph. Explain.

a. $y = 3 - 4x$ and $y = 4x - 3$

b. $y = -6 - 8x$ and $y = 8x - 6$

c. $y = 2x^2$ and $y = 2x(x + 2) - 4x$

d. $y = 5 - x$ and $y = -x - 5$

e. $y = -x + 5$ and $y = 5 - x$

13. Look at your answers to problem 12. For each pair that would not have the same graph, graph both functions on the same axes. Find the point where the two graphs intersect and label it on the graph.

14. Which of the pairs of graphs that you drew in problem 13 do not have a point of intersection? Can you explain why this is so?

15. When graphing two linear functions, there are three possibilities: You may get the same line, two parallel lines, or two lines that intersect. Explain what the tables of (x, y) values look like in each case.

ALWAYS, SOMETIMES, NEVER

While an identity is true for all values of x, an equation may be true for only some values of x, or for no values of x.

Examples: $2x + 6 = 4$ is true when $x = -1$, but not when $x = 0$. The equation $x + 5 = x$ is never true, because a number is never equal to five more than itself. We say this equation has *no solution.*

16. For each equation, state whether it is *always, sometimes,* or *never* true. If it is always or never true, explain how you know. It may help to simplify and to use tables, graphs, or sketches of the Lab Gear.
a. $2x + 5 = 2x + 1$
b. $3(x - 4) - 4(x - 3) = 0$
c. $(x + 5)^2 = x^2 + 25$
d. $6x - (7 - x) + 8 = 7x + 1$

17. Look at the equations in problem 16 that you decided were *sometimes* true. For each one, find a value of x that makes it true and one that makes it false. Show your work.

For each equation 18-21:

State whether the equation is always, sometimes, or never true. Explain.

18. $0.5x - 2 = 0.5(x - 2)$

19. $0.5x - 2 = 0.5(x - 4)$

20. $0.5x - 2 = x - 4$

21. $0.5(x - 2) = x - 4$

22. Report Write a report about equations that are always, sometimes, or never true. Use one example of each type. Illustrate each example with a graph and a Lab Gear sketch. Be sure to include the definition of *identity* and full explanations.

REVIEW **WHICH IS GREATER?**

23. Which is greater, or does it depend on the value of x? Explain.

a.	$-2x$	$-2x + 7$
b.	$6x - 4$	$6x + 4$
c.	$-x^2$	x^2
d.	$(-x)^2$	$-x^2$

REVIEW/PREVIEW **MAKE A SQUARE**

Make a square with these blocks, adding as many yellow blocks as you want, but nothing else. For each square, write an equation relating the side length to the area.

24. $x^2 + 10x +$ __ **25.** $4x^2 + 8x +$ __

26. $9x^2 + 6x +$ __ **27.** $x^2 + 2x +$ __

28. $4x^2 + 12 +$ __

29. Is it possible to get a different square by adding a different number of yellow blocks? Explain your answer.

Make a square with these blocks, adding as many x-blocks as you want, but nothing else. For each square, write an equation relating the side length to the area.

30. $x^2 +$ __ $+ 25$ **31.** $4x^2 +$ __ $+ 25$

32. $x^2 +$ __ $+ 36$ **33.** $9x^2 +$ __ $+ 1$

34. $x^2 +$ __ $+ 9$

35. Is it possible to get a different square by adding a different number of x-blocks? Explain your answer.

36. Summary Describe the pattern for the square of a binomial, in terms of the Lab Gear, and in terms of the algebraic symbols.

THINKING WRITING

6.A Money Matters

DISCOUNTS

A discount card at a movie theater costs \$10. With that card, it costs only \$3 to attend a movie, instead of \$5. The card is valid for three months.

1. Use the same pair of axes for both of the graphs in this problem. Make a graph of the *total cost* (including the cost of the discount card if you got one) as a function of the *number of movies* you see

a. if you have the discount card;

b. if you do not have the discount card.

2. What is the total cost of seeing n movies in three months

a. with the discount card?

b. without the discount card?

3. a. If you saw 12 movies in three months, how much would you save by buying the discount card?

b. If you saw only two movies in three months, how much would you save by *not* buying the discount card?

4. Report Write a report explaining how you would decide whether or not to buy the card. Do a complete analysis of the situation, using graphs, tables, and equations. Your discussion should include, but not be limited to, answers to the following questions:

- What is the break-even point; that is, how many movies would you have to see in order to spend exactly the same amount with and without the discount card?
- How would your decision be affected if the cost of the discount card were raised to \$12?
- How would your decision be affected if the cost of the discount card were changed to $\$K$?

PAYMENTS

Today Lara opened a bank account and deposited \$700. She has just started a part-time job and will get a paycheck of \$130 every two weeks, on the 1st and the 15th of the month. She plans to take \$40 out of each paycheck for cash expenses and deposit the rest in her bank account. On the 15th of every month, when her car payment is due, she will write a check for \$220.

5. Make a table showing how much money Lara will have in her account on the 1st and 15th of every month over the next five months. It may help to show deposits and withdrawals. Look for a pattern.

6. How much money will Lara have in her account on the 1st and the 15th of the month

a. eight months after receiving her first paycheck?

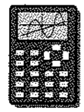
b. n months after receiving her first paycheck?

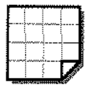
7. Report Imagine that you are Lara's older sister or brother. Write a letter to her showing why she will run out of money, and when. Give her some suggestions for what she might do to avoid this.

LESSON 6.5

Graphical Solutions

You will need:

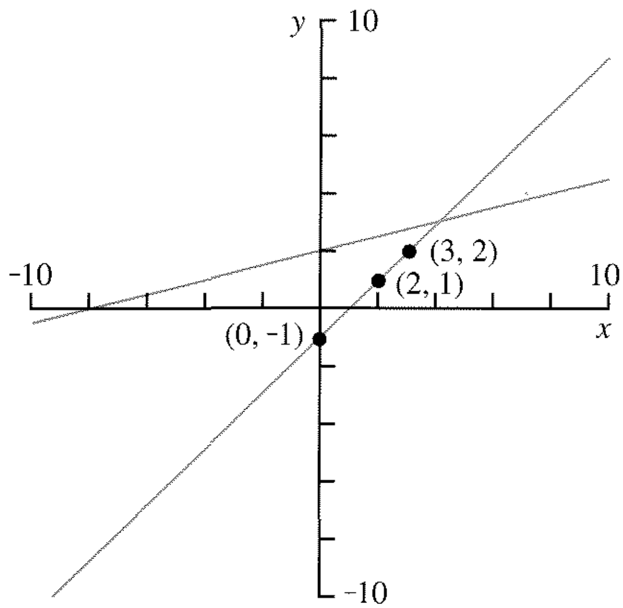
graphing calculator (optional) 

graph paper 

A GRAPHICAL ANALYSIS

1. On the same axes, graph $y = x - 1$ and $y = 0.25x + 2$.

Your graph should look like the one below. The three points that are marked and labeled with their coordinates are all on the part of the graph of $y = x - 1$ that is *below* the graph of $y = 0.25x + 2$.



2. Find the coordinates of three points on the part of the line $y = x - 1$ that is *above* the graph of $y = 0.25x + 2$.

3. Find the coordinates of the point where the two lines cross.

4. If $x = 100$,

a. which graph is above, $y = x - 1$ or $y = 0.25x + 2$?

b. what is the value of $0.25x + 2$?

c. what is the value of $x - 1$?

5. If $x = -100$,

a. which graph is above, $y = x - 1$ or $y = 0.25x + 2$?

b. what is the value of $0.25x + 2$?

c. what is the value of $x - 1$?

6. Describe all the values of x for which the graph of $y = x - 1$

a. is above the graph of $y = 0.25x + 2$.

b. is below the graph of $y = 0.25x + 2$.

7. Describe all the values of x that satisfy each equation or inequality.

a. $0.25x + 2 = x - 1$

b. $0.25x + 2 > x - 1$

c. $0.25x + 2 < x - 1$

FINDING SOLUTIONS

8. Using trial and error, find three values of x that satisfy each inequality.

a. $2x < 3x + 1$

b. $2x > 3x + 1$

It is often easy to find a few values of x that satisfy an inequality. It is harder to find *all* the values, that is, to *solve* the inequality. You have solved equations and inequalities using trial and error, the cover-up method, tables, and the Lab Gear. Another method is to use graphs.

9. Graph $y = 2x$ and $y = 3x + 1$ on the same pair of axes. Use the graphs to solve the two inequalities in problem 8. Remember that even though the graph shows values of both x and y, the original inequalities involved only the variable x. Your answers should involve only x.

10. Graph each pair of functions on graph paper. Use a separate grid for each pair.
    a. $y = 2x - 10$ and $y = 5x - 1$
    b. $y = 2x + 10$ and $y = 5x - 2$
    c. $y = 2x - 10$ and $y = 5x - 2$
    d. $y = x^2$ and $y = 4x - 4$

11. Use your graphs from problem 10 to find the values of x that make these equations true.
    a. $2x - 10 = 5x - 1$
    b. $2x + 10 = 5x - 2$
    c. $2x - 10 = 5x - 2$
    d. $x^2 = 4x - 4$

12. **Summary** Write a paragraph explaining how you can use graphs to help solve equations and inequalities. Illustrate by showing how you would use your method to solve these equations and inequalities.
    a. $-2x + 1 > 3x - 4$
    b. $2x - 1 > -3x + 4$
    c. $3x + 4 = -2x - 6$
    d. $x^2 = x + 2$

MORE EQUATIONS AND INEQUALITIES

Use the techniques you have learned to solve these equations and inequalities. You can use trial and error, the cover-up method, tables, graphs, or the Lab Gear. Show your work.

13. $6x + 1 \leq -3x + 7$

14. $2x + 32 = 6x + 28$

15. $4(x + 5) = 4x + 20$

16. $-3 + m < -m - 3$

17. $\frac{5x + 3}{4} - 6 = 1$

18. $x^2 = 6 - x$

19. $\frac{x}{x + 1} = 1$

20. $\frac{x + 5}{2} + x = 19$

REVIEW SUBSTITUTION

For each problem, write a simple expression that shows the relationship between Δ and $\Diamond$. (Hint: If you cannot find the relationship by using algebra, make a table of values of Δ and $\Diamond$ that make the expressions true, and find a pattern in the table.) Show your work.

21. $\Delta - \Diamond = \Delta$

22. $\Diamond + 2 = \Diamond + \Delta + \Delta$

23. $\Diamond + \Delta + \Delta + \Diamond = \Diamond$

24. $\Diamond - \Delta + \Diamond - \Delta = \Diamond$

25. $\Delta + \Delta = \Diamond + \Diamond$

26. $\Diamond + \Delta + \Delta + \Diamond = 4$

REVIEW/PREVIEW DIVISION AND THE DISTRIBUTIVE LAW

To divide a polynomial by a monomial, you can use the multiplication table format. For example, here is the setup to divide $10x^2 - 5x$ by 5.

	?	?
5	$10x^2$	$-5x$

Ask yourself: *What times* $5 = 10x^2$*?* and *what times* $5 = -5x$*?* Write the answers across the top of the table: $2x^2 - x$.

Divide.

27. $\frac{10x^2 - 5x}{x}$ **28.** $\frac{10x^2 - 5x}{5x}$

If the denominator does not divide every term of the numerator, you will still have fractions in the answer. For example:

$$\frac{10x^2 - 5x}{2} = 5x^2 - \frac{5x}{2}$$

Divide.

29. $\frac{10x^2 - 5x}{10}$ **30.** $\frac{10x^2 - 5x}{x^2}$

31. $\frac{10x^2 - 5x}{3}$

DISCOVERY WEIGHTED AVERAGES

Mr. Cody counts the quiz average (Q) in his class three times as much as the test average (T). That is, he uses the formula:

$$\frac{3Q + T}{4}$$

(This is called a *weighted* average, because he weights the quizzes three times as much.)

Mr. Fletcher counts the test average twice as much as the quiz average. He uses the formula:

$$\frac{Q + 2T}{3}$$

Oliver's grades:
Quizzes: 75 80 85 95 70
Tests: 95 100 80

Connie's grades:
Quizzes: 95 98 94 88 90
Tests: 80 80 95

32. Which teacher would Oliver prefer to have?

33. Which teacher would Connie prefer to have?

34. Oliver and Connie are both in Mr. Dodge's class. He gives students an A who have an average of 90 or better. If possible, show how Mr. Dodge can weight the tests and quizzes so that

a. Oliver has an A average;

b. Connie has an A average;

c. both Connie and Oliver have an A average.

LESSON 6.6

Solving Techniques: Addition and Subtraction

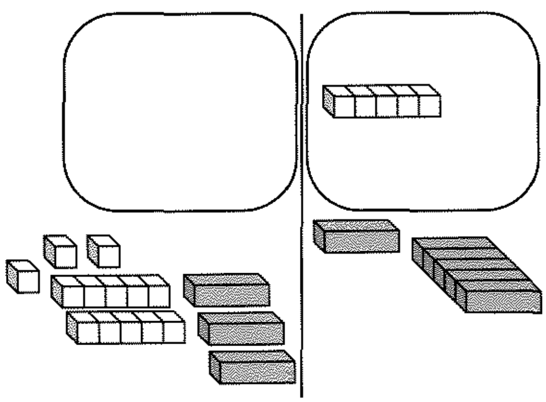
You will need:

the Lab Gear 

One key to solving linear equations is a technique based on this fact: If two quantities are equal, and you **add or subtract the same quantity from both**, you end up with equal quantities. This provides you with a method for simplifying equations.

USING THE LAB GEAR

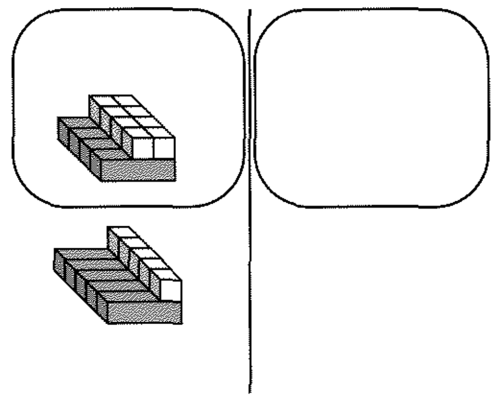
1. Write the equation shown by this figure.



2. Remove three x-blocks from each side. Add 5 to each side and simplify. Finally, form rectangles on both sides, setting them up to show a common side.
3. Write the solution to the equation. Explain.

For each figure in problems 4-7, write the equation, then solve for x. Use the method shown in problem 2.

4.



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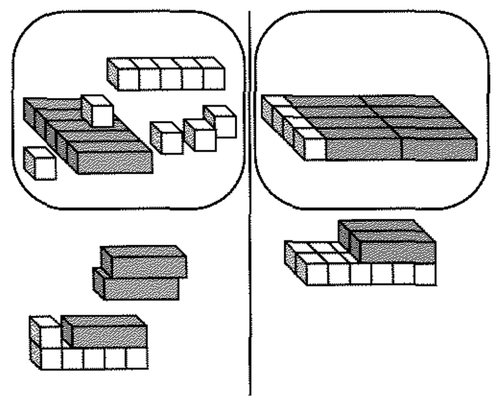

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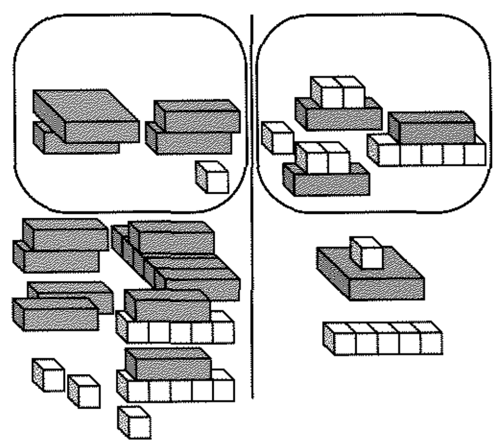

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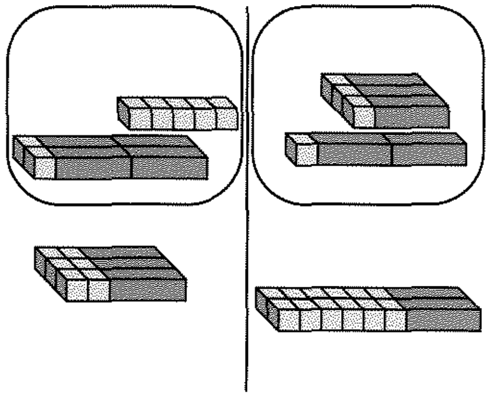


For each problem, 8-11,

a. Model the equation with the Lab Gear.
b. Solve it using the techniques you have learned with the blocks. Record algebraically at least two of the intermediate steps.
c. Write the final answer.

8. $5x + 3(x + 3) = 25$

9. $5x = 25 + 3(x + 3)$

10. $0 = 3(6 - x) + 6x$

11. $15 - 4(x - 2) + 2x = 3$

12. Start with the equation $x = 3$.

a. Add and/or subtract the same amount from both sides repeatedly, getting the equation to be more and more complicated. (The quantities you add and subtract may include x. You may use the Lab Gear. If you do, record some of the steps.) Write the final equation on paper and give it to a classmate.
b. Solve a classmate's equation. If you both do your work correctly, the solution should be $x = 3$. (Again, you may use the Lab Gear.)

SAVINGS PLANS

In this section, you can apply equation solving to real-life problems.

13. Exploration Tania Rhine had \$123. For her birthday, she received \$175 from her grandparents, and her allowance was raised to \$11 a week. What is the largest amount she can spend every week, if

a. she wants to have a total of \$600 by her next birthday?
b. she wants to have \$100 left by her next birthday?

Beatrice had \$321 in her savings account on September 1. She planned to save \$14 a week.

14. Make a table or graph showing how her total savings change as a function of the number of weeks that have passed.

15. Look for a pattern in your table or graph. How much would Bea have at the end of:

a. 4 weeks? b. 52 weeks?
c. n weeks? d. 2 years?
e. n years?

16. Beatrice is considering another possible savings plan. She wants to go to a movie every week, which means she would spend \$5 out of the \$14. She would deposit the rest in her savings account. Make a table or graph of this savings plan to compare with your first one.

17. With the second savings plan, how much would Beatrice have at the end of:

a. 4 weeks?
b. n weeks?

18. Beatrice is saving for a stereo that costs \$549. How long will it take to reach her goal under each savings plan? Try to answer this question without extending your tables or graphs. Instead, try to write and solve equations.

19. Abraham is also saving for the stereo. He has \$235 in his savings account on October 1 and deposits \$21 per week. Write an expression that gives the amount of money that Abraham has after n weeks.

20. Use tables, graphs, or equations to answer these questions. Show your work. Who will have enough to buy the stereo first, Abraham or Beatrice,
   a. if Beatrice has been following her first plan?
   b. if Beatrice has been following her second plan?

21. On January 15, Bea and Abe see an advertisement about the stereo. For two weeks, it will be on sale for \$499. Will either one of them have enough money to buy the stereo then? Do you think one of them will already have bought the stereo? Will your answer depend on what savings plan Bea was following? Explain, showing all your work.

MORE EQUATION SOLVING

Use any of the methods you have learned to solve these equations. Show your work.

22. $3x + 3 - 5x + 6 = 9x - 3x + 23$

23. $5x - 6 = 13x - 5 - 9x$

24. $10x + 23 = 6x + 27$

25. $2 - 3x + 5 = 7x - 4 - 8x$

26. $4x + 5 = 4x + 7$

27. $3x + 4x = 8 + 7x - 8$

DISCOVERY HARDER FACTORING

Factor these trinomials by making a rectangle with the Lab Gear and writing a multiplication equation relating length, width, and area.

28. $x^2 + xy + x + y$

29. $3x^2 + 5x + 2$

30. $6x^2 + 7x + 2$

31. $6x^2 + 19x + 10$

32. $3x^2 + 16x + 5$

33. $4x^2 + 20x + 25$

LESSON 6.7

How Much More Than? How Many Times as Much?

You will need:

- graph paper
- the Lab Gear

COMPARING AGES

On Mark's 12^{th} birthday, he said to his little brother Gordon, "You'd better do what I say. Now I'm twice as old as you are."

The six-year-old math whiz wasn't scared. "That's nothing," he laughed. "A few years ago, you were four times as old as I was. And not long after I was born, you were *thirty-seven* times as old as I was."

1. How old were the two brothers when
   a. Mark was four times as old as Gordon?
   b. Mark was 37 times as old?

2. a. As Mark and Gordon get older, does the *difference* between their ages increase, decrease, or stay the same? Explain.
   b. Does the *ratio* of their ages increase, decrease, or stay the same? Explain.

3. Mark was born in 1980. On the same axes, make two graphs, one showing Mark's age as a function of time and the other showing Gordon's age as a function of time. Label the x-axis *years after 1980* and the y-axis *age*. Compare the two graphs.

4. a. Make a graph showing the *difference* between the two boys' ages as a function of time. Label the x-axis *years after 1980* and the y-axis *difference in ages*. Describe your graph.
   b. Make a graph showing the *ratio* of Mark's age to Gordon's age as a function of time. Label the x-axis *years after 1980* and the y-axis *ratio of ages*. Describe your graph.
   c. Compare the two graphs.

5.
   a. Why do we usually compare people's ages using differences instead of ratios?
   b. What do you think is the smallest possible value for this ratio of Mark's age to Gordon's age? Explain.

6. Beau and Bea said, "The ratio of our ages will always be the same!" How could this be? Discuss.

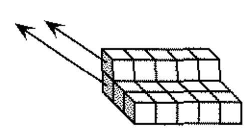
7. On Mark's 12^{th} birthday, his mother was three times as old as Mark. Was she ever twice as old? Was she ever four times as old? Explain.

COMPARING NUMBERS

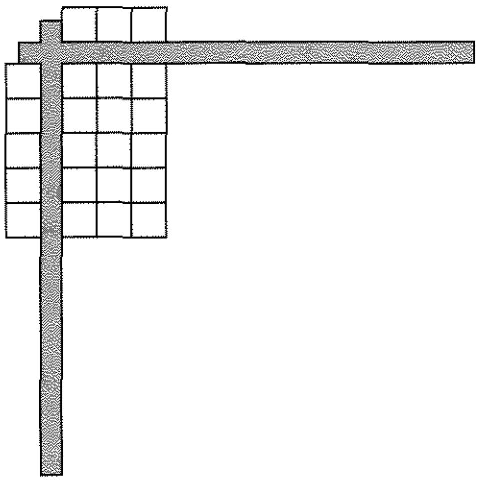
When comparing the size of two positive numbers, for example 5 and 15, you can ask two different questions.

- 15 is *how much more than* 5?
- 15 is *how many times as much* as 5?

The question *How much more than...?* is answered using subtraction, as shown in this figure. Since $15 - 5 = 10$, you can say that 15 is 10 more than 5, (or 10 is the difference of 15 and 5).



The question *How many times as much...?* is answered using division, as shown with the Lab Gear. Since $15/5 = 3$, 15 is 3 times as much as 5, (or 3 is the ratio of 15 and 5).



Answer both questions about these pairs of numbers in problems 8-13. Show how you got your answers. In some cases, you may want to use the Lab Gear.

a. The first number is *how much more than* the second?

b. The first number is *how many times as much* as the second?

8. 35 and 5

9. 10 and 10

10. 9 and 8

11. 16 and 4

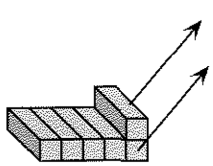
12. 16 and $\frac{1}{4}$

13. 4 and 16

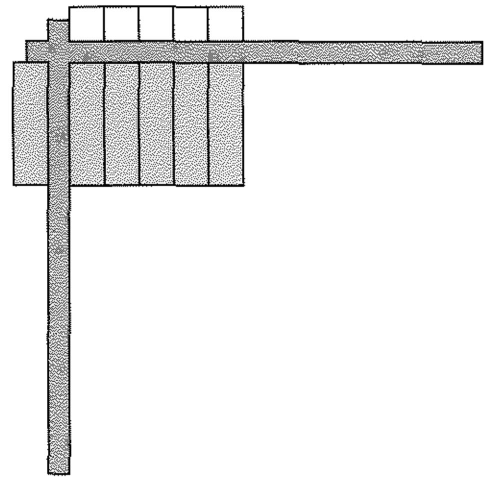
COMPARING ALGEBRAIC EXPRESSIONS

Sometimes you need to compare quantities given by formulas that involve variables. The same methods can be used as when comparing numbers.

To find out *how much more* $5x$ is than x, subtract $5x - x$, as shown.



To find out *how many times as much* $5x$ is than x, divide as shown.



For each pair of expressions in problems 14-18:

a. The first expression is *how much more than* the second?

b. The first expression is *how many times as much* as the second?

14. $5x$ and x

15. $10x$ and 5

16. $10x$ and $5x$

17. $8xy$ and $2x$

18. $2x + 2y$ and $x + y$

APPLICATIONS

19. The Statue of Liberty, which guards the entrance to New York harbor, was given to the United States by the people of France in honor of the centennial of American independence. The statue measures 111 feet 1 inch, from her heel to the top of her head. She was designed by Frederic Auguste Bartholdi. Suppose Mr. Bartholdi had used as a model for the statue a woman who was 5 feet 1 inch tall.

a. How much taller is the statue than the model?

b. How many times as tall is the statue?

c. Which of these two numbers would have been useful to Mr. Bartholdi when designing the statue? Explain.

20. If Reg takes the bus to work, it takes him about an hour and 15 minutes. If he drives, it takes him about 45 minutes.

a. How much longer does it take on the bus?

b. How many times as long does it take?

c. Which number would be more important to Reg in deciding which method of transportation to use? Why?

21. The A.R. Bagel Company charged 30 cents for a bagel in 1973 and 60 cents in 1983. During the same period of time, the hourly wage of a bagel deliverer increased from $2.50 per hour to $5.00 per hour. The company president said, "We try to pay our employees the highest possible wages and charge our customers the lowest possible prices. In a period of high inflation, our prices have risen only 30 cents in ten years. Yet, during the same time, we doubled hourly wages." How might the president of the Bagel Workers' Union describe this situation? Discuss.

DISCOVERY ***TOURNAMENTS***

22. Twelve teams are playing in a tournament.

a. Each team must be scheduled to play three games with each other team. How many games must be scheduled? (Hint: Start by thinking of a smaller tournament.)

b. The teams play "best out of three" games. In other words, the third game of the three may not get played. What is the smallest number of games that might be played?

LESSON 6.8

Solving Techniques: Multiplication and Division

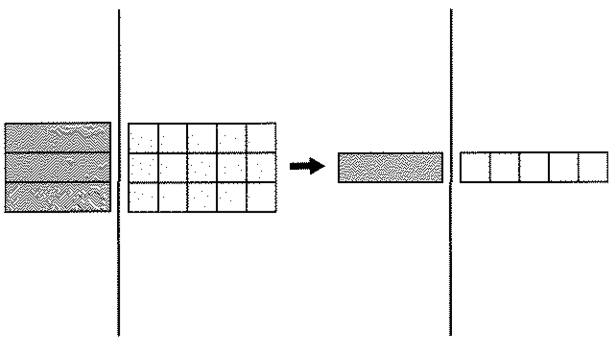
You will need:

the Lab Gear 

graph paper 

Another key to solving equations is the fact that you can *multiply* or *divide both sides by the same number* (as long as it's not zero).

For example, if $3x = 15$, then divide both sides by 3, and you find that $x = 5$.

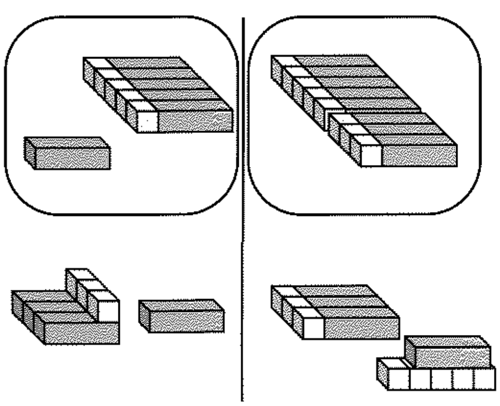


Of course, some divisions cannot be shown easily with the blocks. If you end up with $4y = 7$, then dividing both sides by 4 will reveal that $y = 7/4$. This is impossible to show with the Lab Gear.

USING THE LAB GEAR

Write and solve these equations.

1.



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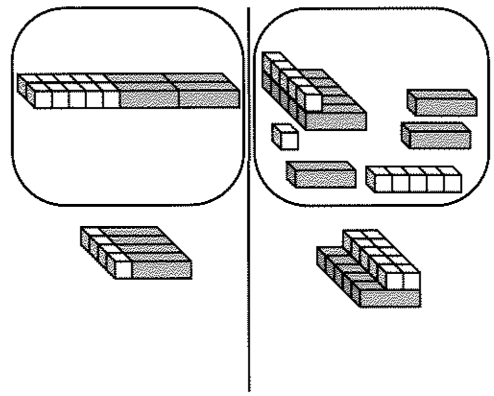

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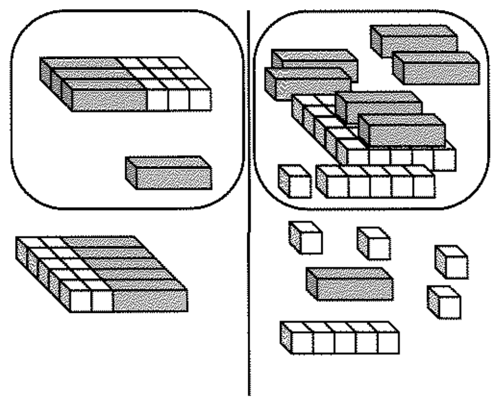
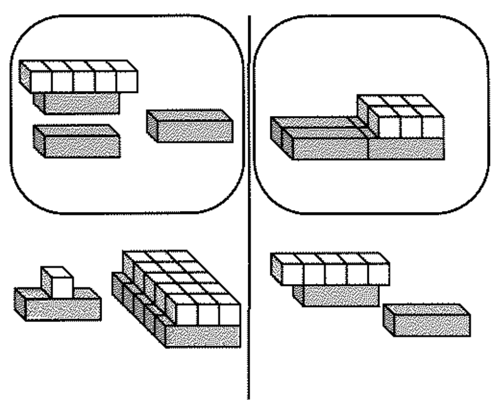


4.



USING RECIPROCALS

Solve these without the Lab Gear.

5. $\frac{2}{3}x = 18$ **6.** $\frac{1}{5}x = 99$

Solve these by multiplying or dividing first, and then again by first distributing the number in front of the parentheses. You should get the same answers by both methods.

7. $7(x - 2) = 30$ **8.** $12(x + 6) = 48$

9. $\frac{1}{3}(2x - 4) = 5$ **10.** $\frac{4}{5}(2 - 8x) = 16$

11. $\frac{1}{2}(2x - 4) = 5$ **12.** $\frac{5}{4}(2 - 8x) = 16$

13. Summary Use examples.

a. Explain how to decide which of the two methods (distributing first or later) one should use in problems 7-12.

b. Explain how to decide what number to multiply or divide both sides by when solving an equation.

14. Start with $x = -3$.

a. Create an equation by adding and/or subtracting the same amount from both sides repeatedly, and by multiplying and/or dividing both sides by the same amount repeatedly. Write the final equation on paper and give it to a classmate.

b. Solve a classmate's equation. If you both do your work correctly, the solution should be -3.

SOLVING FOR y

You have learned to multiply or divide by a number when solving an equation containing one variable. This is also a useful technique when working with equations containing two variables, such as this one, $4y - 8x = 0$.

In a two-variable equation, it is often useful to solve for one variable *in terms of* another. This means that one variable is alone on one side of the equation.

By adding $8x$ to both sides, it is easy to rewrite this equation so that the y's are on one side and the x's are on the other:

$$4y = 8x.$$

Dividing both sides by 4 gives

$$y = 2x.$$

Transform each equation below so that y is in terms of x. You may use the Lab Gear.

15. $3y - 6x = 9$ **16.** $6x - 3y = 12$

17. $x - y = 1$ **18.** $6x - 5y = 0$

EQUIVALENT EQUATIONS

19. Draw axes and plot three (x, y) pairs that satisfy the graph of $4y - 8x = 0$. Describe the graph.

20. Find three (x, y) pairs that satisfy $y = 2x$ and draw the graph. Compare it with the graph in problem 19. What do you notice? Explain.

If equations in two variables have the same graph on the Cartesian coordinate system, they are called *equivalent equations.*

21. Explain how you could have determined *without graphing* that the equation $4y - 8x = 0$ is equivalent to $y = 2x$.

22. a. Write an equation that is equivalent to $6y = 12x$, but looks different.

b. Describe what the graphs of both equations would look like.

For each group of equations decide which ones, if any, are equivalent equations. If you are unsure, you might want to solve the equations for y, make some tables, or draw some graphs.

23. $x + y = 2$
$2x + 2y = 2$
$2x + 2y = 4$

24. $x/y = 12$
$y/x = 12$
$y = 12x$

25. $3x - y = 6$
$2y = 6x - 12$
$y - 3x = 6$

26. $0.8x = y$
$x - 0.2x = y$
$y - 4/5x = 0$

27. $1.2\,x = y$
$x + 0.2x - y = 0$
$2.4x - 2y - x = 0$

PUZZLES ***AGE RIDDLES***

28. At age 3, Henry could count to 12. How far could he count by age 21?

29. Augustus De Morgan lived in the nineteenth century. He said, "I was x years old in the year x^2." In what year was he born?

30. Diophantus spent one-sixth of his life in childhood and one-twelfth of his life in youth. He spent one-seventh more of his life as a bachelor. Five years after he was married, his son was born. His son lived half as long as his father and died four years before his father. How many years did Diophantus live? How old was he when he got married?

31. Make up an age riddle.

32. Solve a classmate's riddle.

RESEARCH ***FAMOUS MATHEMATICIANS***

33. Prepare a report about Diophantus or Augustus De Morgan. What were their contributions to mathematics?

THINKING WRITING

6.B Constant Differences, Constant Ratios

You will need:

graph paper

CONSTANT DIFFERENCE GRAPHS

These three (x, y) pairs follow a pattern: (6, 0), (7, 1), (-4, -10). The difference between y and x always equals 6. The equation $y - x = -6$ describes the relationship between x and y.

Use the same pair of axes for all the graphs in problems 1-5.

1. Graph $y - x = -6$.
2. Choose any other integer D and graph the function $y - x = D$. (For example, if you chose the integer 10, you would graph the equation $y - x = 10$.) Label the graph with its equation.
3. Graph several other functions of the form $y - x = D$. For each graph, you will need to choose a different number for D. Remember to try negative numbers and fractions as well as positive integers.
4. Compare your *constant difference* graphs with the *constant sum* graphs that you investigated in Chapter 5, Lesson 1.
5. Graph some constant difference graphs of the form $x - y = D$. Explain any differences or similarities with graphs of the form $y - x = D$.

CONSTANT RATIO GRAPHS

These three (x, y) pairs follow a pattern: (3, 6), (4, 8), (-4, -8). The ratio of y to x is always equal to 2. The equation $y/x = 2$ describes the relationship between x and y.

Use the same pair of axes for the graphs in problems 6-9.

6. Graph $y/x = 2$.
7. Choose any other number R and graph the function $y/x = R$. (For example, if you chose R to be 3, you would graph the equation $y/x = 3$.) Label the graph with its equation.
8. Graph several other functions of the form $y/x = R$. For each graph, you will need to choose a different number for R. Be sure to try some negative and fractional values as well as positive integers.
9. Now graph some *constant ratio* graphs of the form $x/y = R$. Explain any differences and similarities with graphs of the form $y/x=R$.

PARAMETERS

Note: D and R in problems 2-9 are called *parameters.*

10. **Report** Write a report describing and analyzing any patterns you noticed in the graphs you just drew. Your report should be divided into two parts, one on constant differences, and the other on constant ratios. It should include, but not be limited to, answers to these questions:
    - Can you tell from the value of the parameter which quadrants the lines will pass through? Whether the lines slope up or down?
    - Do any lines go through the origin? If not, do you think you could find a value for the parameter so that the line would go through the origin? Explain.
    - For the constant ratio graphs, why is there a "hole" in the graph when $x = 0$?
    - Comment on anything you notice about the x-intercepts and y-intercepts.
    - There is one constant difference graph that is also a constant ratio graph. What are the values of D and R?

LESSON 6.9

Rational Expressions

You will need:

the Lab Gear 

COMPARING RATIONAL EXPRESSIONS

To add, subtract, multiply, and divide fractions involving variables, use the same rules you use for numerical fractions.

1. Review the rules for adding, subtracting, multiplying, and dividing fractions, using an example of each kind.

2. For each expression, substitute 1, 2, and 9 for x and perform the indicated operation. In which problem is the answer the same, regardless of the value of x?

   a. $\frac{5}{x} \cdot \frac{x}{5}$ b. $\left(\frac{5}{x}\right) / \left(\frac{x}{5}\right)$

   c. $\frac{5}{x} + \frac{x}{5}$ d. $\frac{5}{x} - \frac{x}{5}$

A *rational number* is any number that can be written as a ratio of integers. A *rational expression* is an expression that involves a *ratio.* A very simple rational expression is the rational number 1/2, which is the ratio of 1 to 2. A more complicated rational expression is $(x^2 + 3x + 4)/(x^3 - 99)$, which is the ratio of two polynomials.

3. With the numbers 3 and 4, you can write the ratio 3/4 or the ratio 4/3.
   a. Which is greater, 3/4 or 4/3?
   b. Which is greater, 3/4 of 4/3 or 4/3 of 3/4? Explain.

4. For each pair of expressions below, write:

   A if the expression in the first column is greater

   B if the expression in the second column is greater

   ? if the value of x determines which one is greater

   Explain your answers.

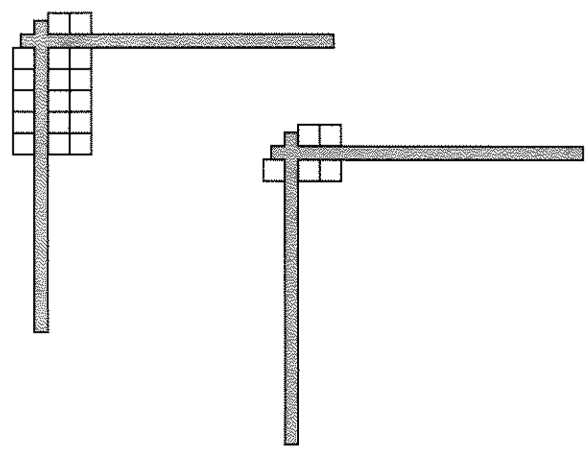
   a. $\frac{x}{5}$ $\frac{x-2}{5}$

   b. $x - \frac{2}{5}$ $\frac{x-2}{5}$

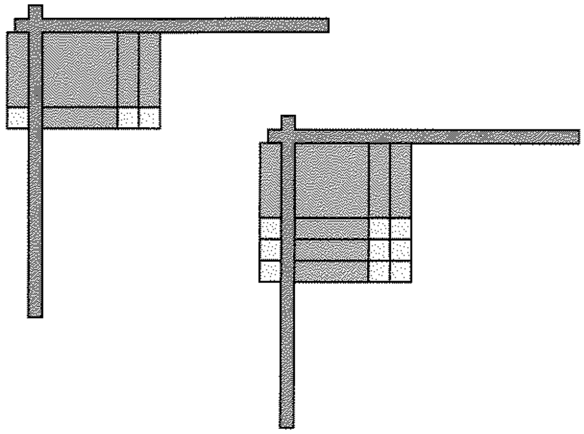
   c. $\frac{5}{x}$ $\frac{5}{x-2}$

EQUIVALENT RATIONAL EXPRESSIONS

By dividing, you can show that two fractions represent the same ratio. For example, as the figure shows, 10/5 equals 2/1.



The same thing sometimes works with polynomials. As shown in the figure, the rational expression $(x^2 + 3x + 2)/(x + 1)$ is equal to $(x^2 + 5x + 6)/(x + 3)$ because the result of both divisions is the same.



5. What is the result of both divisions?

(**Note:** These two rational expressions are **not** equal when $x = -1$ or $x = -3$. Can you see why? Try substituting these numbers for x and see what happens.)

6. Are these rational expressions equal? Explain. You may use a sketch.

$(xy + 2x)/x \qquad (y^2 + 2y)/y$

7. For each problem, find a number or expression you could put in the box that would make the two rational expressions equal. Explain each part, perhaps using a sketch.

a. $\frac{3x}{x} \qquad \frac{3x+6}{\square}$

b. $\frac{18}{6} \qquad \frac{15}{\square}$

c. $\frac{\square}{x+y} \qquad \frac{2x+2y}{2}$

d. $\frac{x^2+8x+12}{\square} \qquad \frac{2x+12}{2}$

8. **Exploration** The equation

$$\frac{x-3}{5} = x + 1$$

cannot easily be modeled with the Lab Gear. Try to solve it using any technique you have learned. Compare your method and your answers with other students' work.

Lea and Earl both tried to solve this equation. They got different answers.

Lea's work	**Earl's work**
$5\left(\frac{x-3}{5}\right) = (x+1)\cdot 5$	$5\left(\frac{x-3}{5}\right) = (x+1)\cdot 5$
$x - 3 = 5x + 5$	$5(x-3) = 5(x+1)$
$-3 = 4x + 5$	$\frac{1}{5}\cdot 5(x-3) = \frac{1}{5}\cdot 5(x+1)$
$-2 = 4x$	$x - 3 = x + 1$
$-0.5 = x$	impossible

9. Who is right, or are they both wrong? Copy each student's work and write an explanation beside each step telling what was done. If a step is incorrect, explain why and make a correction.

Solve these equations. Show your work and write a brief explanation of each step.

10. $\frac{6-x}{8} = \frac{x}{2}$

11. $\frac{6-x}{8} = \frac{x}{5}$

12. $\frac{x+5}{x+7} = 3$

13. $\frac{6+2x}{x+7} = \frac{4}{5}$

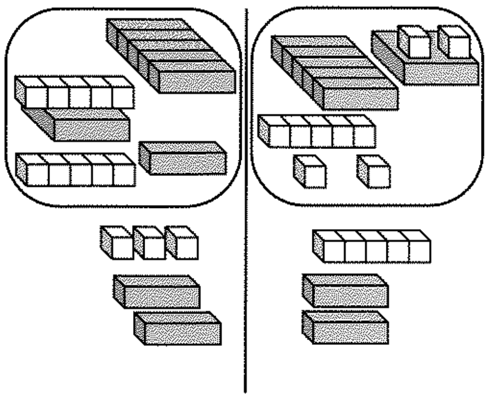
14. $\frac{x-3}{2} = x + 5$

15. **Summary** Describe a general method for solving equations like those in problems 10-14. Include the solution of an example you made up.

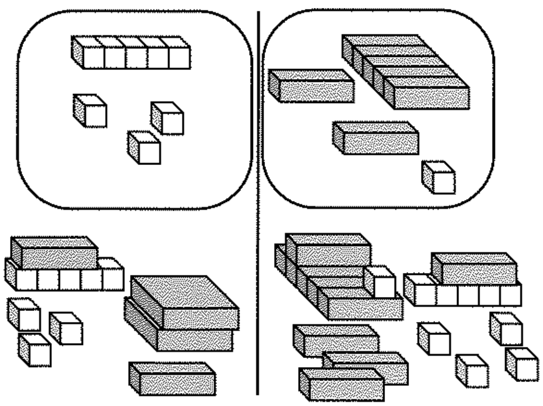
REVIEW EQUATION SOLVING

Write the equation shown by the blocks, then solve it. If you use the Lab Gear, write equations to show some of the steps as you move your blocks. If you don't, show all your work.

16.



17.



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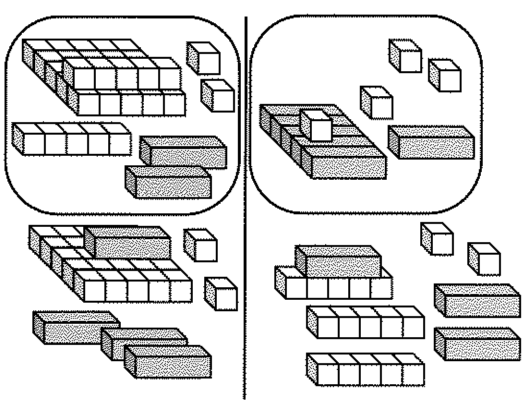
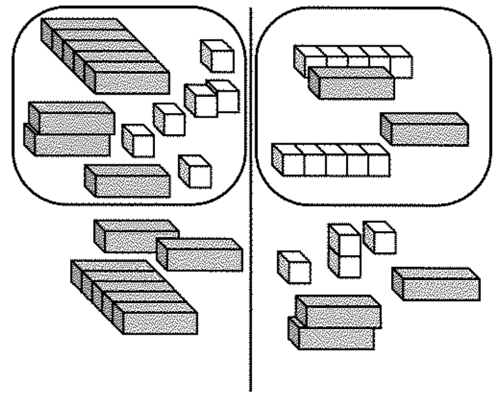


19.



Solve.

20. $(x + 3)^2 = (x - 3)(x + 4)$

21. $(x - 1)^2 = (x + 2)(x - 6)$

PUZZLES ORDER OF OPERATIONS

Keeping in mind order of operations, insert as many pairs of parentheses as needed, to make these equations true.

22. $4 \cdot 2 + 3 = 20$

23. $\frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

24. $5 \cdot 3 - 2 + 6 = 35$

25. $3^2 + 2 \cdot 7 - 4 = 33$

26. $\frac{1}{3} \cdot 6 + 4 \cdot \frac{2}{6} + \frac{1}{3} = \frac{7}{3}$

27. $1 - 2 \cdot 2 + 5 \cdot 6 = -42$

LESSON 6.10

Improving Your Average

FREE THROWS

Alaberg High School has a girls' basketball team nicknamed "the Gals." Ms. Ball, the coach, is studying these statistics.

Mid-Season Free-Throw Data

	FT-A	Average
Bea	15-20	75%
Gale	3-18	____
Lara	____-8	25%
Lea	5-____	20%
Li Ann	16-24	____

FT-A means *free throws made - free throws attempted.* The average is shown as a percent, but it could be shown as a ratio or decimal. (For example, Bea's average is 15/20 or 0.75.)

1. Copy and complete the table. Who has the best record so far this season?

2. Exploration Bea wants to have a season record of 90%. She thinks she can make every free throw that she attempts for the rest of the season. Tell how many she would have to make in a row in order for her season average to be:

a. 80% b. 85% c. 90%
d. 95% e. 99% f. 100%

Discuss.

3. If Bea has had 20 free throw attempts and has made 15 of them, her average is 15/20. If she has x more attempts and makes all of them, her average is $\frac{15 + x}{20 + x}$.

a. What is the value of this ratio when $x = 40$? (That is, what is her average if she has 40 more attempts and makes all of them?)

b. What is her average if $x = 25$?

4. Suppose Li Ann had x free throw attempts during the rest of the season and *missed* every one.

a. What would her season average be, in terms of x? (Hint: The expression will be different from the one in problem 3.)

b. If she had a season average of 40%, how many more free throws after mid-season must she have attempted?

c. If she attempted ten more free throws, what would her season average be?

5. Suppose Li Ann *made* every attempted free throw.

a. What would her season average be, in terms of x?

b. What would her season average be if she attempted eight more free throws?

c. If she had a season average of 0.85, how many more free throws must she have attempted?

These problems are not very realistic. Usually people do not make all their attempted free throws, but they don't miss all of them either. Lea hopes that she will make about 40% of her attempted free throws for the rest of the season.

6. If Lea attempts x more free throws, and makes 40% of them, she knows that her average for the season would be

$$\frac{5 + 0.40x}{25 + x}.$$

a. Explain the meaning of the numerator and denominator of this expression, and how it was figured out.
b. How would the expression change if Lea made 60% of her remaining free throws?
c. How would the expression change if Lea made 20% of her remaining free throws?

7. Assume Lea makes 40% of her remaining free throws and wants to raise her season average to at least 30%. What is the minimum number of free throws she needs?

8. By the end of the season Gale had doubled both her attempts and her successes. What happened to her average?

9. **Generalization** Assume a student has made M out of T free throws. Assume she attempts x more shots and makes N of them. What will her season average be in terms of M, T, x, and N? Explain.

GRADES

Alaberg High School has a "no pass, no play" rule for all sports. Students must have an average of 65% in all their classes in order to qualify to play any sport the following quarter.

Some members of the boys' basketball team (the Bears) are worrying about their averages for algebra. (See the table.)

Their grades in algebra are based on 12 ten-point assignments per quarter. Students who have been absent because of illness (like Hal and Zal) can complete the assignments late.

Mid-Quarter Algebra Scores

	Possible points	Points earned	Average
Al	80	35	—
Hal	70	52	—
Cal	80	63	—
Zal	60	59	—
Sid	80	74	—

10. Copy and complete the table.

Use the table to answer the following questions. Assume that *passing* means having an average of 65% or better, and *failing* means having an average below 65%.

11. Who has the lowest average so far?

12. Answer the following questions for each student.
   a. What is the worst conceivable average he could get by the end of the course?
   b. What is the best conceivable average he could get?
   c. What is the smallest number of points he needs to earn in the remaining assignments in order to pass?

REVIEW *EQUATION SOLVING*

Solve for the variable.

13. $\frac{y+5}{2} = \frac{19-y}{1}$

14. $\frac{y+5}{4} = \frac{19-y}{2}$

15. $2(y + 5) = 19 - y$

16. $4(y + 5) = 2(19 - y)$

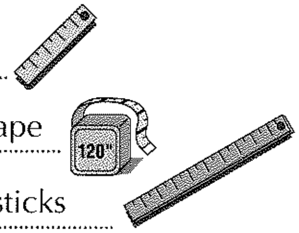
17. $y + 5 = 2(19 - y)$

18. $\frac{-15 + 3x}{5 + 4x} = 7$

Stuart Little and Alice

You will need:

- rulers
- measuring tape
- and/or yardsticks



STUART LITTLE

Here is the beginning of *Stuart Little,* a children's book by E.B. White.

> When Mrs. Frederick C. Little's second son arrived, everybody noticed that he was not much bigger than a mouse. The truth of the matter was, the baby looked very much like a mouse in every way. He was only about two inches high; and he had a mouse's sharp nose, a mouse's tail, a mouse's whiskers, and the pleasant, shy manner of a mouse. Before he was many days old he was not only looking like a mouse but acting like one, too — wearing a gray hat and carrying a small cane. Mr. and Mrs. Little named him Stuart, and Mr. Little made him a tiny bed out of four clothespins and a cigarette box.

1. Measure, in inches, the height of several boys in your class. To do the following exercises, choose someone whose height is near the average of the heights you measured.
2. Measure, in inches, the length and width of the average boy's
   a. pants; b. shirt or coat.
3. Measure, in inches, the length and width of:
   a. a book or binder;
   b. a chair or desk.
4. Calculate the size of each item in problems 2-3, if it were to be made for Stuart Little. Explain your work.
5. Draw each item in the size that you calculated in problem 4.

ALICE

Here is an excerpt from *Alice in Wonderland,* a book by the English mathematician Lewis Carroll.

> ...this bottle was not marked "poison," so Alice ventured to taste it, and finding it very nice, (it had, in fact, a sort of mixed flavour of cherry-tart, custard, pine-apple, roast turkey, toffy, and hot buttered toast), she very soon finished it off.
>
> * * * * * * * * * * * * * *
>
> "What a curious feeling!" said Alice, "I must be shutting up like a telescope!"
>
> And so it was indeed: she was now only ten inches high, and her face brightened up at the thought that she was now the right size for going through the little door into that lovely garden.

6. Measure, in inches, the height of several girls in your class. To do the following exercises, choose someone whose height is near the average of the heights you measured.
7. Assuming that before she drank from the bottle, Alice was the size of the average girl in your class, how many times as tall was she after shrinking?
8. a. Measure a real pencil or pen.
   b. Calculate the correct size for a pencil or pen of the same kind for Alice. Explain.

c. Draw it in the size you calculated in part (b).

9. Measure a real door, and calculate the dimensions of "the little door into that lovely garden."

> "Curiouser and curiouser!" cried Alice (she was so much surprised, that for the moment she quite forgot how to speak good English). "Now I'm opening out like the largest telescope that ever was! Goodbye, feet!"...
>
> ...Just at this moment, her head struck against the roof of the hall: in fact she was now rather more than nine feet high...

10. How many times as tall as an average girl in your class is Alice now?

11. What would be the size of a pencil if it were the right size for giant Alice? Show your calculations.

THE BIG FRIENDLY GIANT

The following are quotations about the Big Friendly Giant, a character in Roald Dahl's book *The BFG.*

a. It was four times as tall as the tallest human.

b. It actually had to bend down to peer into the upstairs windows. That's how tall it was.

c. ...an arm as thick as a tree trunk...

d. The Giant was sprinting down the High Street... Each stride he took was as long as a tennis court.

e. In the middle of the floor there was a table twelve feet high...

f. He had truly enormous ears. Each one was as big as the wheel of a truck...

12. Project Estimate the height of the Giant using the information given in each quotation. Explain your work.

- What real-world numbers did you use?
- How did you find them?
- What calculations did you do?
- Did the results of your calculations agree with each other?
- Based on all the calculations, what is your final estimate of the Giant's height?

YOUR OWN STORY

13. Project

a. Write and illustrate a story for a young child featuring little people or giants. Make sure the dimensions of all objects are sized correctly.

b. On a separate piece of paper, explain your calculations.

OTHER STORIES

14. Project Ask a librarian or an elementary school teacher to suggest a book that involves little people or giants. Make up math problems based on the book. Use specific quotations from the book as much as possible. On a separate piece of paper, solve the problems you make up.

REVIEW SOLVING EQUATIONS

15. Solve the equation,

$$2.5x + 18 + 1.5x - 11 = 19.$$

16. If $x = 3$, calculate, $2.5x + 18 + 1.5x - 11$.

17. Explain how problems 15 and 16 are related.

LESSON 6.12

Geoboard Squares

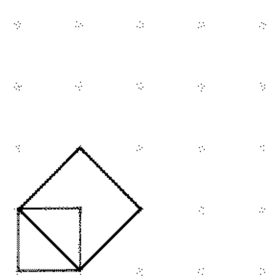
You will need:
- geoboards
- dot paper

1. **Exploration** There are 33 different geoboard squares. Find as many of them as you can. (For this exercise, squares that have the same size are considered the same.) Sketch each square on dot paper.

FIND THE AREA

2. There are 10 geoboard squares having horizontal and vertical sides. What are their areas?

3. Make a 1-by-1 square in the bottom left of your geoboard. Make a square that has this square's diagonal — (0, 1) to (1, 0) — for a side. What is the area of the new square?



4. Repeat problem 3, starting with larger and larger squares in the bottom left. What is the area of each new square?

5. Explain why only five squares can be found this way.

6. Make a square having (0, 1) to (2, 0) as a side.

a. Explain how you found the other vertices of the square.

b. Find the area. Explain how you did it.

7. Make squares having (0, 1) to (x, 0) as a side. Use x = 3, 4, ... 9. Find the area of each one.

8. Explain why you cannot find a geoboard square having (0, 1) to (10, 0) as one side.

9. Make a square having (0, 2) to (3, 0) as one side.

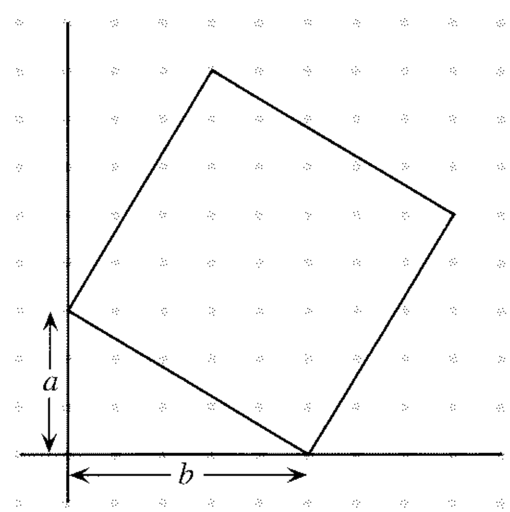
a. Sketch the square.

b. Make and sketch the smallest square having horizontal and vertical sides that entirely covers the original square. What is the area of this square?

c. What is the total area of the four triangles that surround the original square?

d. What is the area of the original square?

10. **Generalization** On dot paper, sketch a pair of x- and y-axes.



a. Copy the above figure.

b. Sketch the smallest square having horizontal and vertical sides that entirely covers the original square. What are its sides in terms of a and b? What is its area in terms of a and b?

c. What is the area of one of the triangles that surround the original square in terms of a and b? What is the total area of the four triangles in terms of a and b?

d. What is the area of the original square in terms of a and b?

11. **Summary** How does one find the area of a geoboard square? For examples, use the squares having the following as one side.

a. (0, 3) to (4, 0)

b. (2, 4) to (7, 1)

FIND THE SIDE

12. How long is the side of a square if the area is

a. 64? b. 81?

c. 289? d. 0.0121?

13. How long is the side of a square if the area is 70? Give an approximate answer. (Hint: You may be able to use some of the results from problem 12.)

14. Use trial and error on a calculator to answer problem 13 to the closest one-thousandth.

15. **Summary** Use examples to explain.

a. How does one find the area of a square, if given the side?

b. How does one find the side of a square, if given the area?

DISCOVERY CHUNKING

16. Solve: $3(x + 3) + 5 = 4(x + 3)$

It is easier to solve this equation by first solving for $(x + 3)$, and then finding x, instead of distributing. This is called *chunking*, since in this method the quantity $(x + 3)$ is thought of as one chunk.

17. Solve $3y + 5 = 4y$, then use the fact that $y = x + 3$ to solve for x. Explain what you did, and how this problem is related to problem 16.

18. Create an equation that would be easier to solve by chunking than by distributing. Solve it.

19. Solve a classmate's chunking equation.

DISCOVERY INEQUALITY RULES

Like most students, Mary and Martin enjoy discussing inequalities during their lunch period.

20. Martin said, "I noticed something cool. If $5/x$ is less than 5, then $x/5$ is more than 1/5." Mary said, "I don't understand. In the first place, I can't think of a value of x that would make $5/x$ greater than 5."

a. Give Mary at least two values of x that will make $5/x$ greater than 5.

b. Is Martin's statement correct? Give examples to explain your answer.

21. Martin said, "If $a < b$, then by taking the reciprocals of both sides, I get $1/a > 1/b$. Notice that I changed the direction of the inequality." Mary answered, "Sorry, but you're wrong." Who is right? Explain, with examples.

22. Martin said, "If $a < b$, then by taking the opposites of both sides, I get $-a > -b$. Notice that I changed the direction of the inequality." Mary answered, "When will you give up making up rules off the top of your head! You're wrong again!" Who is right? Explain, with examples.

REVIEW SOLVING INEQUALITIES

Solve these inequalities. Remember that you must find *all the values* of x that make the inequality true. Show your work, and check your answers.

23. $x - 1 > 5$

24. $x + 1 > 5$

25. $2x - 6 > 5x + 3$

26. $2x - 6 < 5x + 3$

27. $3(x + 1) > 6$

28. $2 - 3(x + 1) > 6$

DISCOVERY CAN TARA MAKE A B?

Some auto insurance policies have a "good student" policy for high school students. If a student maintains a *B* average, he or she can qualify for a discount on insurance rates.

Tara doesn't like writing assignments because they take time outside of school, when she would rather be driving her car. However, she does well on quizzes. She needs a *B* in algebra. Her scores are:

Writing Assignments: 45 55
Quizzes: 100 50 90 85 90 95

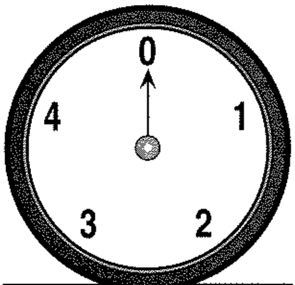
Tara hopes that the teacher will count quizzes heavily in the average so that she can make a *B*.

29. Is it possible for Tara to make a *B*? If so, how much would the teacher have to weight her quizzes? If not, explain why not.

THINKING WRITING

6.C Group Theory

MOD CLOCKS



Mod 5 Clock

The figure shows a mod clock, which is a special function machine. For any positive whole number input, it will output a number between 0 and 4. For example:

Input	Output	Input	Output
1	1	5	0
9	4	12	2
13	3	17	2
25	0	26	1
77	2	100	0

1. What would be the output of the mod clock for the following inputs? Explain.
a. 1998 b. 1899 c. 9981

Definition: $a \oplus b$ is the output from the mod clock for the input $a + b$. $a \otimes b$ is the output for the input ab.

Example: $3 \oplus 2 = 0$, and $3 \otimes 2 = 1$

2. Make a table for each of $\oplus$ and $\otimes$.

3. Generalization The clock above is a mod 5 clock. Find ways to predict the output of mod 10, mod 2, mod 9, and mod 3 clocks.

GROUPS

Definition: A *group* is a set of elements, together with an operation that satisfies the following rules.

- *closure:* using the operation on two elements of the group yields an element of the group.
- *associative law:* $(ab)c = a(bc)$.
- *identity element:* one of the elements, e, is such that $ae = ea = a$, for any element a in the group.
- *inverse element:* every element a has an inverse a' such that $a\,a' = a'a = e$

Some groups are *commutative* ($ab = ba$) and some are not.

For 4-7 assume the associative law holds.

4. a. Show that the set $\{0, 1, 2, 3, 4\}$ together with the operation $\oplus$ is a group.
b. Show that $\{0, 1, 2, 3, 4\}$ with $\otimes$ is not a group.
c. Show that $\{1, 2, 3, 4\}$ with $\otimes$ is a group.

5. Is the set of the integers a group with the following operations?
a. addition b. multiplication

6. Show that the set of rational numbers (positive and negative fractions and zero) together with multiplication is not a group. By removing one element, it can be made into a group. Which element? Explain.

7. Think about a mod 4 clock, with the numbers $\{0, 1, 2, 3\}$. Is it a group for $\oplus$? For $\otimes$? Can it be made into one by removing an element?

8. Report Give examples of groups. For each, give the set and operation. Explain how they satisfy the rules. Include finite, infinite, commutative, and noncommutative groups.

Essential Ideas

EQUATIONS, IDENTITIES, INEQUALITIES

1. Always, sometimes, or never true?
   a. $2x + 6 = 2x - 6$
   b. $2x + 6 = 2(x + 6)$
   c. $2x + 6 = x + 6$
   d. $2x + 6 = 2(x + 3)$

2. For each equation above, decide which of the two expressions is greater, if they are equal, or if the answer depends on the value of x.

3. Solve the inequalities. You may want to use a graph.
   a. $3x < 5$ b. $x + 3 < 5$
   c. $3x + 3 < 5$ d. $2x + 6 < x + 6$

SOLVING EQUATIONS

Solve these equations.

4. a. $4x + 8 = 9$
   b. $-4x + 8 = 9$
   c. $4x - 8 = 9$
   d. $-4x + 8 = -9$

5. a. $x - 6 = 2(x - 5)$
   b. $2x - 12 = 4(x - 5)$
   c. $2.5(x - 5) = 2.5x - 12$

6. a. $\frac{1}{3}(4x - 2) = 5$
   b. $\frac{4}{5}(8 - 2x) = 16$
   c. $\frac{x - 3}{2} = x - 4.3$

7. a. $6 - 3(m - 4) = 3m$
   b. $(6 - 3)(n - 4) = 3n$
   c. $6 - 2(p + 4) = (8 - p)(2 + 3)$
   d. $(6 - x)(x + 4) = (8 - x)(x + 2)$

8. a. $\frac{d + 9}{5} - 3 = 15$ b. $\frac{2d + 6}{5} = \frac{3d - 7}{5}$
   c. $\frac{f - 2}{4} = f + 3$

9. Solve for y in terms of x.
   a. $-6x + y = 4$
   b. $2y + x = 8$

GRAPHS

10. Graph these equations on the same axes.
   $y - x = -6$ $y = 2(x - 5)$
   $y = 2x - 12$ $y = 4(x - 5)$

11. Explain how one can use this graph to check the solutions to problem 5.

12. Use your graph to solve the compound inequality, $2x - 12 < x - 6 < 2x - 10$. Explain.

WRITING EQUATIONS

13. Write an expression telling how much money Bea will have if she
   a. starts with \$321 and saves \$9 a week for n weeks;
   b. starts with \$321 and saves \$$d$ a week for n weeks;
   c. starts with \$$m$ and saves \$$d$ a week for n weeks.

14. If Bea starts with \$321, how much must she save each week to reach \$456 in 28 weeks? Write an equation and solve it.

DIFFERENCES AND RATIOS

According to author Glen Rounds, Johnny Inkslinger was Paul Bunyan's accountant. He used a pencil that was "over three feet in diameter and seventy-six feet long — the first one ever used." A typical pencil is a quarter inch in diameter and seven and a half inches long. Most men in those days were probably between 5 feet 6 in. and 6 feet tall.

15. Compared to a normal pencil, Johnny Inkslinger's was
   a. how much wider?
   b. how many times as wide?
   c. how much longer?
   d. how many times as long?

16. Based on this information, how tall do you think Johnny was? Explain. (Give your answer as a range of probable heights.)

TABLES, GRAPHS, AND EQUATIONS

A telephone company offers two different billing plans. The Community Plan costs \$10.77 a month and allows unlimited local calls. The Thrifty Plan costs \$5.50 a month, but the cost of local calls is 5.5 cents for the first minute, plus 3.5 cents for each additional minute. Both plans cost the same for long distance calls. Which plan should different callers use?

17. Assume that your phone calls last an average of five minutes.
   a. How much does an average call cost under the Thrifty Plan?
   b. Write a formula for the Thrifty Plan. Use y for the cost, x for the number of phone calls.
   c. If you make exactly one five-minute call a day, should you use the Thrifty Plan or the Community Plan?

18. Write a formula for the Thrifty Plan. Use y for the cost and x for the number of phone calls. Assume your calls last an average of:
   a. 1 minute; b. 3 minutes;
   c. 5 minutes; d. 7 minutes.

19. Make tables to show how many calls a month make it preferable to use the Community Plan, for a customer whose calls last an average of:
   a. 1 minute; b. 3 minutes;
   c. 5 minutes; d. 7 minutes.

20. Use a graph to show the costs of both plans for each customer listed in problem 19 as a function of the number of calls made. (Your graph should include five lines.)

A consumer advocate gives advice to people about which plan to choose. In order to do that, he needs to generalize the information revealed in problems 17-20.

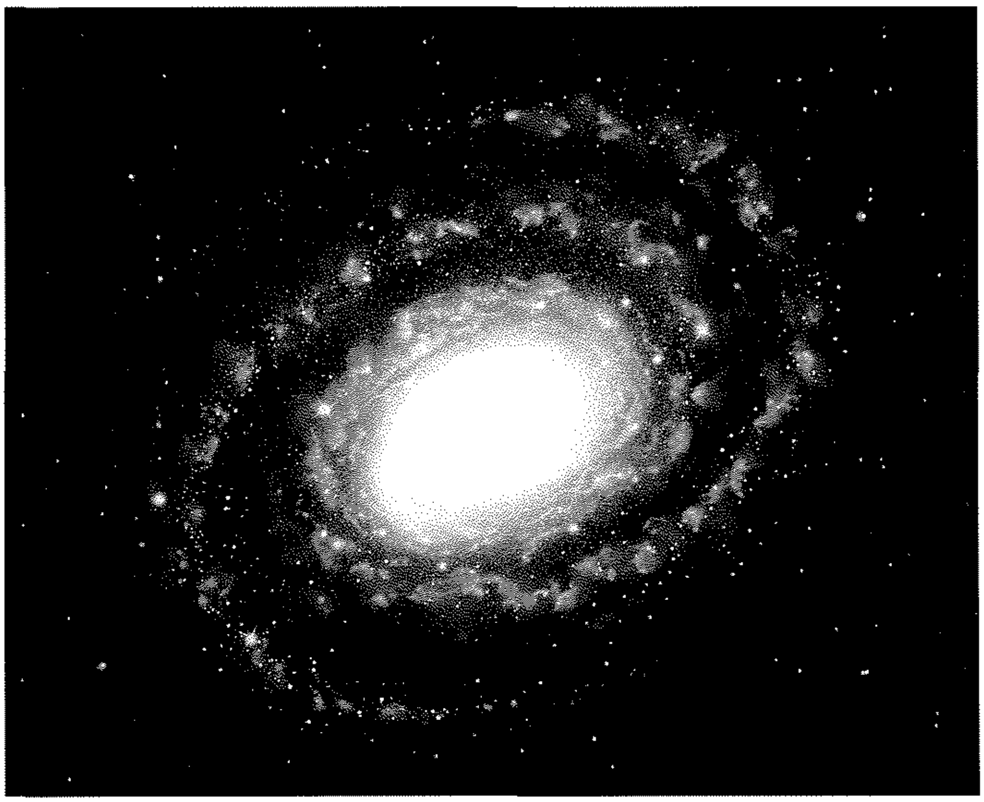
21. He would like to have a formula for the Thrifty Plan in terms of two variables: x for the number of local calls, and t for the average duration of each call. Find such a formula.

22. He would like to know the number of local calls at the "break even" point, where both plans cost roughly the same amount, in terms of t. To figure this out, he sets up an equation, with the formula for the Thrifty Plan on the left, and the cost of the Community Plan (10.77) on the right.
   a. Solve the equation for x.
   b. Check your answers to problem 17 with the formula you found in part (a).

23. In trying to use the formula from problem 22 he finds that people don't usually know the average duration of their phone calls. To help them figure it out, he asks them for an estimate of the numbers of local calls they make every week that last approximately: one minute, five minutes, ten minutes, and thirty minutes. Given these four numbers, how can he find the average duration of the phone calls?

24. **Project** Keep track of the duration of your phone calls for a week. Figure out which plan would be more suitable for you if you had your own phone.

CHAPTER 7



A spiral galaxy, having arms made of gas, dust, and stars

Coming in this chapter:

Exploration The expression $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \ldots + n^3$ can be modeled by building n cubes out of blocks. Could you rearrange these blocks into a square? If so, what are its dimensions? Experiment with different values of n. Look for a pattern.