LESSON

1.6 Coming to Terms

You will need:

the Lab Gear 

1. Name a Lab Gear block that can be used as a model for an object with:

a. three dimensions;

b. two dimensions;

c. one dimension;

d. zero dimensions.

Definitions: In the expression

$$x^3 + 2xy - 3x + 4,$$

four quantities are added or subtracted, so we say that there are four *terms:* x^3, $2xy$, $3x$, and 4. Note that a term is a product of numbers and variables. The sum or difference of one or more terms is called a *polynomial.*

Note that polynomials do not involve division by variables. For example, $(1/x) + x$ is not a polynomial.

DEGREE

The *degree* of an expression, in terms of the Lab Gear, is the **lowest dimension** in which you can arrange the blocks. For example, take the expression $3x$. These blocks can be arranged in a rectangle (two dimensions) or in a line (one dimension).

The lowest dimension is one, so the degree of $3x$ is one.

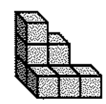
2. Show how the term $2xy$ could be arranged as a box (three dimensions) or as a rectangle (two dimensions). What is the degree of $2xy$?

Of course, x^3 cannot be shown in less than three dimensions, so its degree is 3.

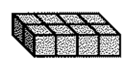
3. Write the degree of each blue block.

The degree of a constant expression (any combination of yellow blocks) is considered to be 0. The reason for this is that the yellow blocks can be separated into 1-blocks, which model zero-dimensional points, with no length, width, or height. See the figure below, which shows how the number 8 can be shown in three ways.

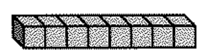
Three dimensions

 or 

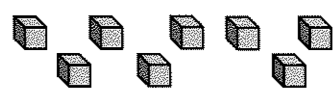
Two dimensions

 or 

One dimension

 or 

Zero dimensions

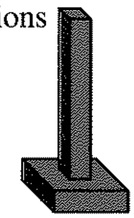


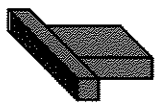
4. What is the degree of these terms?

a. $4y$ b. $5x^2$

c. $2xy^2$ d. 7

The degree of a polynomial can be found in the same way. For example, the figures below show how the blocks x^2 and y can be arranged in figures of two or three dimensions. However note that they cannot be arranged into figures of zero or one dimension.

Three dimensions 

Two dimensions 

5. What is the degree of $x^2 + y$?

6. What is the degree of these polynomials?

a. $4y + 3$ b. $x^3 + 5x^2$
c. $2xy^2 + x^2$ d. $xy + 7$

Definition: The 2 in the term $2xy$ is called the *coefficient*. A term like x^3 has an invisible coefficient, a 1, since $1x^3$ is usually written just x^3.

7. **Generalizations** If two terms differ only by their coefficients (like $2x$ and $5x$) what can you say about their degrees?

8. How can you find the degree of a term without using the Lab Gear? Explain, using examples.

9. How can you find the degree of a polynomial without using the Lab Gear? Explain, using examples.

HIGHER DEGREE

10. Why is it impossible to show $x^2 \cdot x^2$ with the Lab Gear?

11. What is the product of x^2 and x^2?

12. Even though there are only three dimensions in space, terms can be of degree 4. Write as many different terms of degree 4 as you can, using 1 for the coefficient and x and y for the variables.

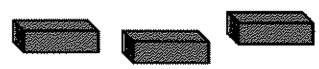
13. Which of these expressions cannot be shown with the blocks? Explain.

a. $5x^2$ b. $2x^5$ c. $\frac{2}{x^5}$ d. $\frac{5}{x^2}$

COMBINING LIKE TERMS

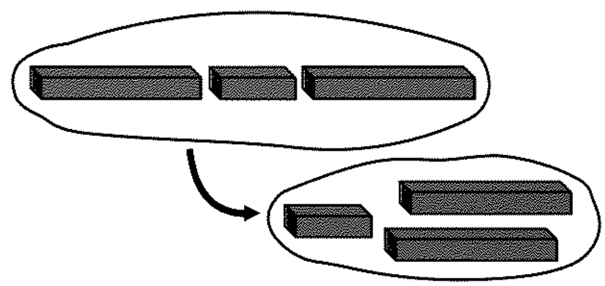
There are many ways you can write an expression that names a collection of Lab Gear blocks. When you put blocks of the same size and shape together and name them according to the arrangement, you are combining *like terms*. Look at these examples.

This quantity is written $x + x + x$,

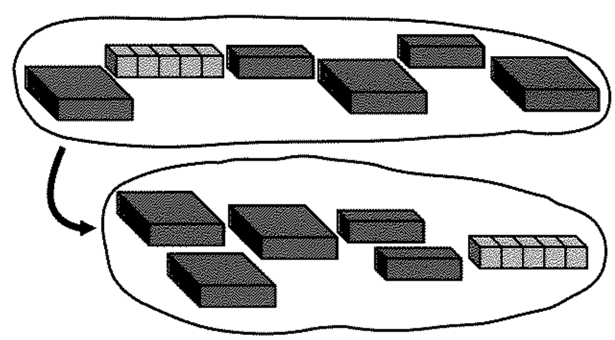


or $3x$, after combining like terms.

This quantity is written $y + x + y$,



or $x + 2y$, after combining like terms.
This quantity is written
$x^2 + 5 + x + x^2 + x + x^2$,

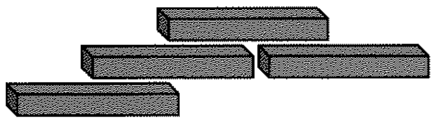


or $3x^2 + 2x + 5$, after combining like terms.

Of course, a $5x$-block, when combining like terms, is equivalent to 5 separate x-blocks. For example, it can be combined with two x-blocks to make $7x$.

For each example, show the figure with your blocks, combine like terms, then write the quantity the short way.

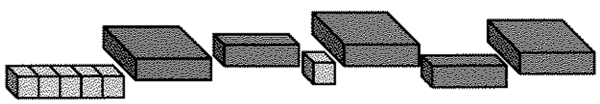
14.



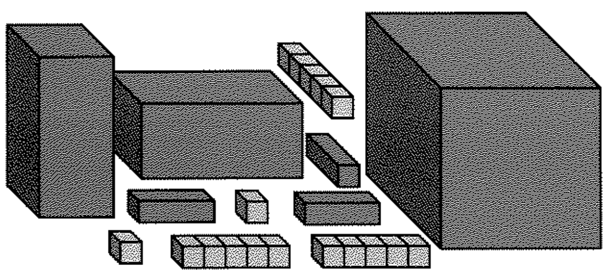
15.

16.

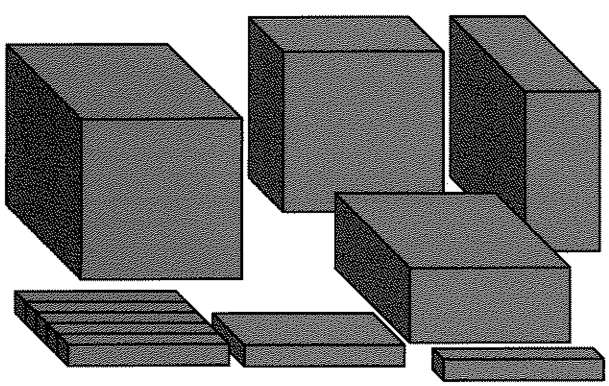
17.



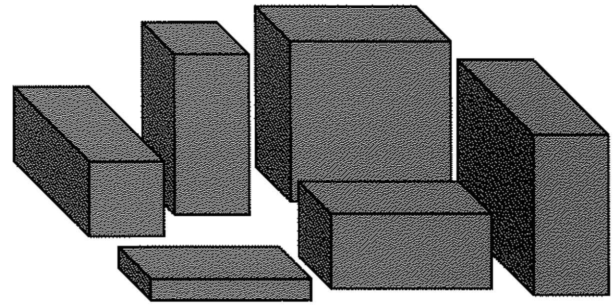
18.



19.



20.



21. What terms are missing? (More than one term is missing in each problem.)

a. $3x^2 + 4x + \_\_\_ = 9x^2 + 8x + 7$

b. $x^2y + 6xy + \_\_\_ = 9x^2y + 8xy$

22. **Summary** Explain, with examples, the words *degree, coefficient, polynomial,* and *like terms*. Use sketches of the Lab Gear as well as explanations in words and symbols.