LESSON 3.11

Math on Another Planet

SMALL POCKETS

On the treeless planet of Glosia, the currency consists of florins, ecus, and ducats. *One florin is worth two ecus, and one ecu is worth two ducats.* Since there is no paper, there is no paper money, and the people of Glosia have to carry coins everywhere. King Evariste VII, being immensely rich, must wear bloomers with enormous reinforced pockets to hold his money.

One day the King realizes that there is a new trend in Glosian fashion. Elegant men and women wear only small pockets. Evariste VII, not one to be left behind by the great movements of style, decides to institute a drastic economic reform by enacting a strange law: *One ducat is worth two florins!* (The old rules are not changed.) When you realize trades can be made in either direction, you can see how the King's brilliant legislation will abolish poverty forever.

The people of Glosia are ecstatic. With the new system, one may have a fortune in one's pockets, and yet never carry more than three coins! One can be rich and fashionable at the same time. For example, if you own eight ecus, you can go to the bank, and trade them in for four florins. These can be traded again, for two ducats, which equal one ecu, which will certainly fit in your pocket.

1. Exploration

a. The King trades his coins at the bank, according to their official value, with the object of having as few coins as possible in the tiny pocket of his slinky new pants. He starts with 1000 florins. What does he end up with?

b. Prince Enbel has one ducat. He buys a toastereo (a popular appliance which, unfortunately, does not make coffee), costing 50 ecus. If he is given the fewest coins possible, how much change does he get?

c. Princess Lisa has one ecu. She wins the first prize in a contest in *Names* Magazine. The prize is one ducat, one ecu, and one florin. She now has four coins, but they won't fit into her pocket. What does she have after trading them in to get as few coins as possible? (The second prize would have been a T-shirt with the *Names* logo and no pockets at all.)

d. Sol Grundy has no money. He gets a job at the toastereo store, earning one florin per day, seven days a week. Since his pockets are fashionably small, he trades his money as often as possible in order to have as few coins as possible. If he starts his new job on Monday, how much does he have each day of the week? The next week? (Assume he doesn't spend any money.)

2. Make a list of the amounts of money one can have that cannot be reduced to a smaller number of coins. (Hint: There are seven possible amounts.) One of the amounts is $(d + e)$.

3. Make an addition table for Glosian money. It should be a seven-by-seven table, with a row and column for each of the amounts you found in problem 2. For example, your table should show that $(d + e) + d = f$.

4. One of the seven amounts you found in problem 3 can be considered to be the "zero" of Glosian money, since adding it to a collection of coins does not change the collection's value (after trading to get the smallest possible number of coins). Which amount is the zero for Glosian money?

5. The opposite of an amount is the amount you add to it to get the zero. Find the opposite of each of the seven amounts in problem 3.

A LONG MONTH

The King can never remember which month it is and how many days the month has. He decides to start a new calendar, with a single infinite month, the month of *Evary,* named after himself. This is what the calendar looks like.

Evary

Mo	**Tu**	**We**	**Th**	**Fr**	**Sa**	**Su**
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	32
33	34	35	36	37	38	...

6. What day of the week will it be on Evary 100th? Explain how you figured it out.

The King is so pleased with the new calendar that he decides to invent a new kind of math. He calls it *Calendar Math.* In Calendar Math, Monday + Tuesday →

$$5 + 6 = 11 \rightarrow \text{Sunday},$$

or, more briefly, Mo + Tu = Su.

7. Check whether, if you picked different numbers for Monday (such as 12, 19, etc.) and Tuesday (13, 20, etc.), you would still get Sunday for the sum.

8. Make an addition table for Calendar Math. It should be a seven-by-seven table, with the days of the week along the left side and across the top and their sums inside the table.

9. *Calendar Zero* is a day of the week such that, when you add it to any other day, you get that other day for the answer. What day is Calendar Zero?

10. Find the *Calendar Opposite* for each day of the week. That is the day you add to a given day to get Calendar Zero. If a day does not have an opposite, or is its own opposite, explain.

11. Calculate.

a. Mo + Mo

b. Mo + Mo + Mo

c. Mo + Mo + Mo + Mo, etc.

12. How many times do you add Mo to itself to get back Mo?

13. Make a multiplication table for Calendar Math. Here is an example of a result that would appear in it.

Mo · Tu → 5 · 6 = 30 → Fr,

so, Mo · Tu = Fr.

14. What is special about Calendar Zero in multiplication?

15. *Calendar One* is a day of the week such that when you multiply it by any other day, you get that other day for the answer. What day is Calendar One?

16. The *Calendar Reciprocal* of a day is the day you multiply it by to get Calendar One. Find the Calendar Reciprocal for each day. If a day does not have a reciprocal, or is its own reciprocal, explain.

17. Calculate Su^2, Su^3, etc. What power of Su is equal to Su?

18. Summary Summarize Calendar Math.