LESSON 4.8

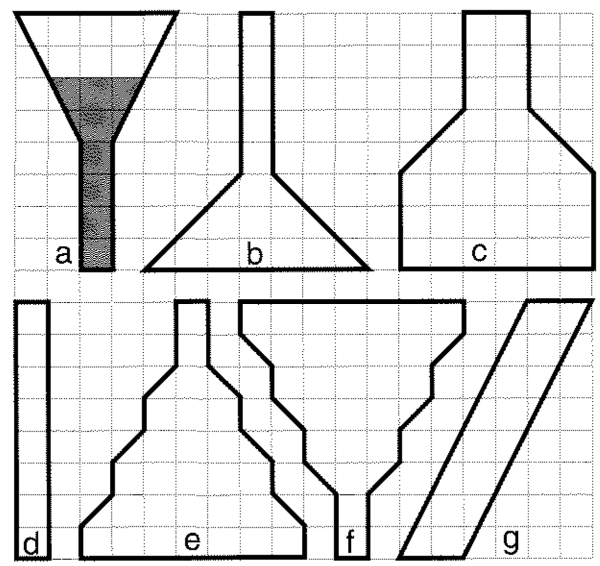
Jarring Discoveries

You will need:

- centimeter graph paper
- jar lids
- string
- centimeter rulers

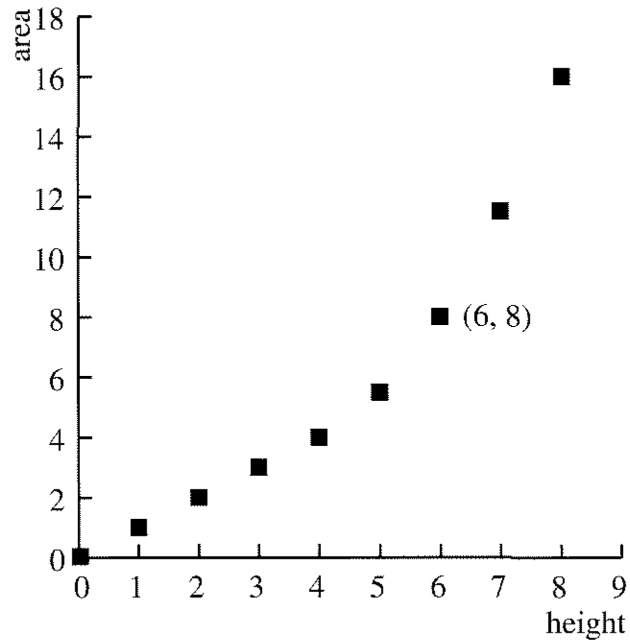
FLAT SCIENCE

Doctor Dimension is a flat scientist. He stores two-dimensional liquids in two-dimensional jars, like the ones shown in this figure.



One day, as part of his scientific research, he decides to graph the amount of liquid in a jar as a function of the height of liquid. Since he lives in a two-dimensional world, liquid is measured in square units. For example, jar (a) is filled to a height of six units and contains eight square units of liquid.

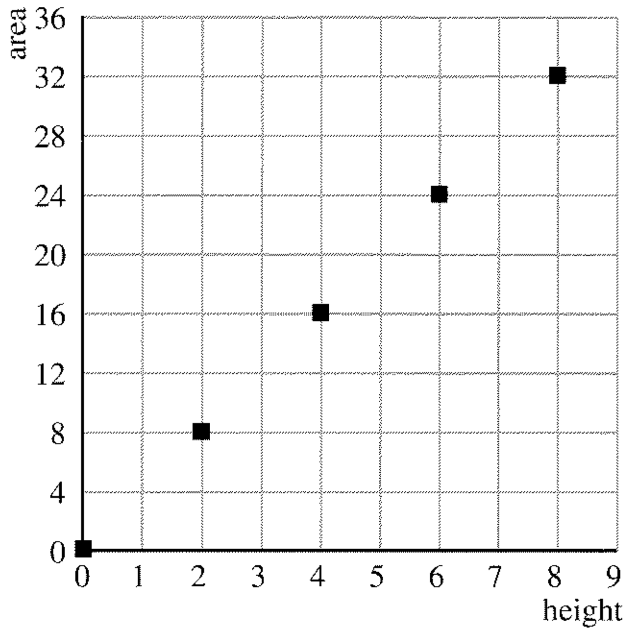
The following graph represents jar (a).



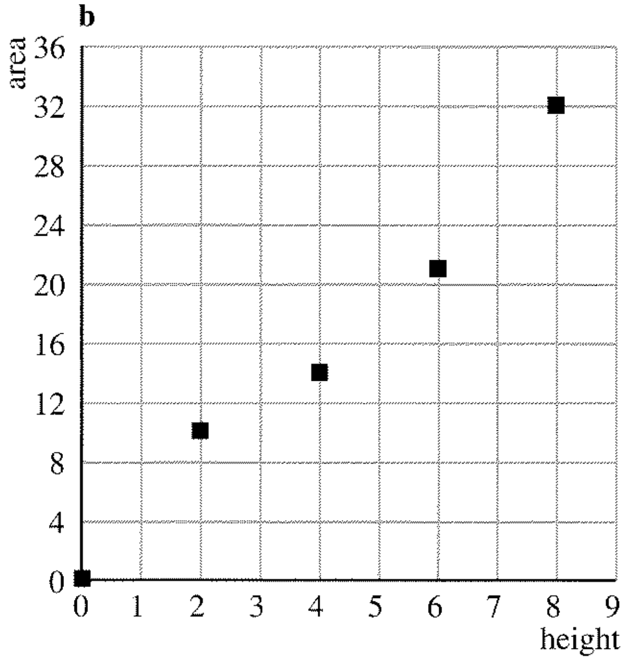
1. Some of the dots lie on one straight line. In which part of the graph does this happen? Explain why this is so.
2. Make a graph for each of the remaining jars.
3. For which jars is the area of liquid a direct variation function of the height? Explain.

4. Draw two different jars for each graph below.

a

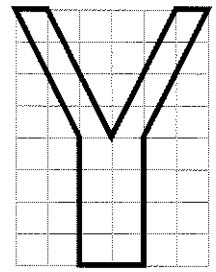


b



5. Draw a jar for part (a) of problem 4 for which the area of liquid is *not* a direct variation function of the height.

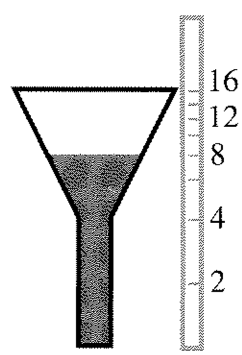
6. Predict the shape of the graph for this jar. Then test your prediction.



7. **Summary** Explain how the shape of the jar affects the shape of the graph. Explain what it takes for a jar to have a graph that is a straight line through the origin.

8. **Generalization** How do you think the shape of a three-dimensional jar affects the shape of the graph of the volume of liquid as a function of height? What jar shapes correspond to a direct variation function?

A dipstick can be used to measure the amount of liquid in a jar, but the dipstick must be specially designed for the jar. For example, the following dipstick would work for jar (a).



On it, area is marked off with a tick for every two square units.

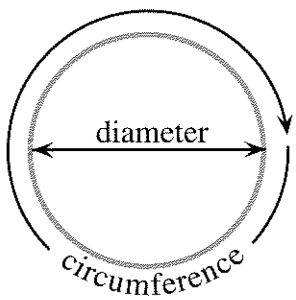
9. Note that the dipstick ticks are not evenly spaced. Explain why.

10. Which jars would have a dipstick whose ticks are evenly spaced? Explain.

11. Project Draw an accurate dipstick for each of several different jars. Write a report showing sketches of the jars and their dipsticks, and explain your method.

JAR LIDS: CIRCUMFERENCE

For this section, use jar lids of at least five different sizes, including one very small one and one very large one.



12. Measure the diameter and circumference of each of the jar lids in centimeters, as accurately as possible. (Use the string to help find the circumference.) Make a table showing your data.

13. Make a graph of your data, putting diameter on the x-axis and circumference on the y-axis. Don't forget to include a point for a lid having diameter 0.

14. What is the relationship of circumference to diameter for each jar lid? Describe it in words and with an equation. Explain how you figured it out.

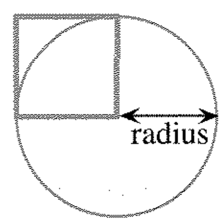
15. Is the relationship between diameter and circumference an example of direct variation? Explain.

16. According to your data, what is the approximate value of the ratio of circumference to diameter?

JAR LIDS: AREA

17. Estimate the area of the top of each jar lid by tracing around it on centimeter graph paper and estimating the number of square centimeters it covers. Make a table and a graph of the relationship between diameter and area, including a point for a lid having diameter 0.

18. Is the relationship between diameter and area an example of direct variation? Explain.



The figure shows a square whose side equals the radius of the circle.

19. For each jar lid, calculate the area of a square like the one shown in the figure. Add a column for these data in your jar-lid area table.

20. Graph the area of the circles as a function of the area of the squares.

21. What is the relationship between the area of the circles and the area of the squares? Describe it in words and with an equation. Explain how you figured it out.

22. Is the relationship between the area of the circles and the area of the squares an example of direct variation? Explain.

23. According to your data, what is the approximate value of the ratio of the area of the circle to the area of the square?

24. Summary According to your data, what is the relationship between the area of a circle and its radius? The area of a circle and its diameter? Explain.

REVIEW *DIVIDING ON A CALCULATOR*

Phil used his calculator to find the reciprocal of 7, and got the number 0.1428571429. Lyn's calculator, on the other hand, gave the number 0.1428571428.

25. Explain how two calculators can give different results, even though neither is defective.

Phil's grandfather does not believe in calculators. He said, "Do you really believe either number is the reciprocal of 7? I have news for you. Multiply each one by 7 without a calculator, and you'll see why you should not trust these machines."

26. Work with a classmate. Do the two multiplications on paper to see who was right, Phil, Lyn, or their grandfather. Explain your results.

The grandfather added, "To find the real reciprocal of 7, you have to use good old-fashioned long division."

27. Find the real reciprocal of 7.

28. Report Write a letter to Lyn and Phil's grandfather, explaining why students are allowed to use calculators nowadays. Your letter should include, but not be limited to:
   - Answers to the grandfather's probable objections;
   - A table showing the real reciprocals of the whole numbers from 0 to 10, and the reciprocals as given by Lyn's and Phil's calculators;
   - An explanation of how you can find the real reciprocal by using a calculator;
   - An argument explaining why Lyn's or Phil's calculator is the better one for the purpose of finding reciprocals.

29. Make a *division table* like this one. Extend it to show whole-number numerators and denominators from 0 to 10. You may use a calculator, but enter only exact answers. Look for patterns and work with a partner. Some answers were entered for you.

Denominators \ Numerators	0	1	2	3	...
0					
1					
2	0	0.5	1	1.5	...
3					
...					...

30. What patterns do you notice about the row of your table for denominator 7?

31. Learn how to use the FIX mode on your calculator.