LESSON 7.7

Equations With Squares

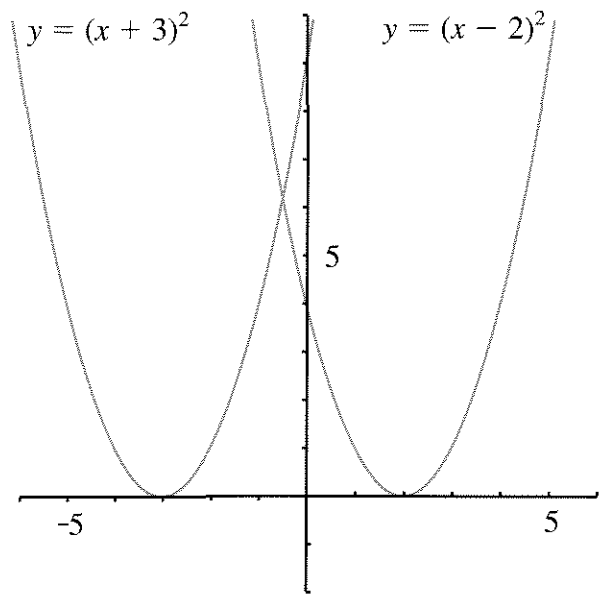
You will need:

- graph paper
- the Lab Gear

This lesson is about solving equations. You will use two different methods to approach equations that involve the square of a binomial.

GRAPHICAL SOLUTIONS

The graphs of $y = (x + 3)^2$ and $y = (x - 2)^2$ are shown below.



1. Explain why these two graphs never go below the x-axis. (Why is the value of y never negative?)

2. On a piece of graph paper, copy the two graphs. For more accuracy, calculate the coordinates of several points on each curve. Use the graphs to solve these equations.

a. $(x + 3)^2 = 4$
b. $(x - 2)^2 = 9$
c. $(x - 2)^2 = 1$
d. $(x + 3)^2 = -1$
e. $(x - 2)^2 = 0$

3. Use your graphs to estimate the solutions to these equations.

a. $(x + 3)^2 = 12$
b. $(x - 2)^2 = 6$
c. $(x - 2)^2 = -2$
d. $(x + 3)^2 = 5$
e. $(x + 3)^2 = (x - 2)^2$

4.

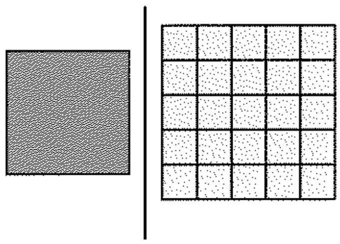
a. Describe what you think the graphs of the functions $y = (x + 2)^2$ and $y = (x - 1)^2$ would look like. (Where would each one intersect the x-axis?)
b. Check your guess by making tables of values and graphing the functions.

5. Use your graphs to find or estimate the solutions to these equations.

a. $(x + 2)^2 = 9$
b. $(x + 2)^2 = 2x + 3$
c. $(x - 1)^2 = 5$
d. $(x - 1)^2 = -x$

EQUAL SQUARES

The equation $x^2 = 25$ can be illustrated using the Lab Gear. Put out your blocks like this.

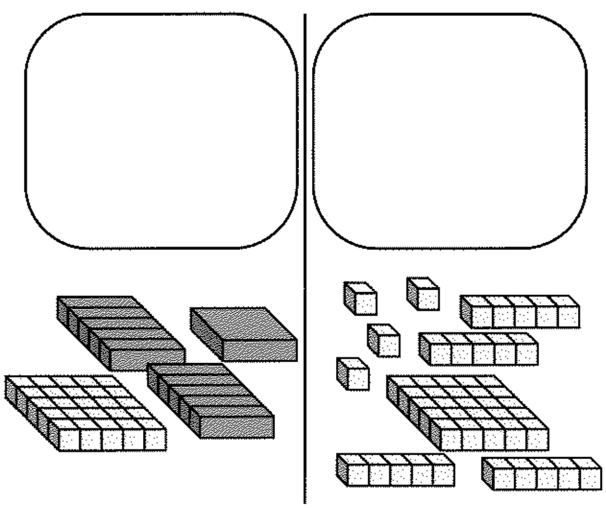


One way to get started with this equation is to remember that if the squares are equal, *their sides must be equal.* (This is true even though they don't look equal. Remember that x can have any value.)

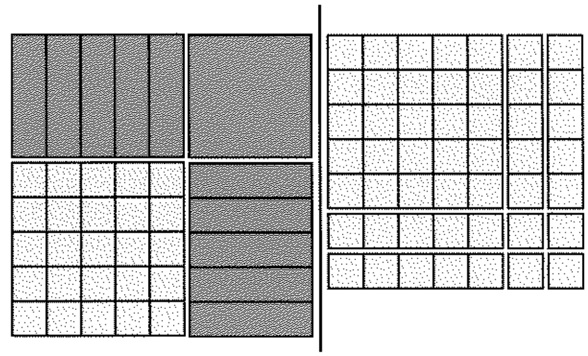
6. Solve the equation. If you found only one solution, think some more, because there are two.

7. Explain why there are two solutions.

8. Write the equation shown by this figure.



By rearranging the blocks, you can see that this is an *equal squares* problem, so it can be solved the same way. As you can see, $(x + 5)^2 = 7^2$. It follows that $x + 5 = 7$ or $x + 5 = -7$.



9. Solve the equation. There are two solutions. Check them both in the original equation.

Solve the equations 10-13 using the *equal squares* method. You do not have to use the actual blocks, but you can if you want to. Most equations, but not all, have two solutions.

10. $x^2 = 16$

11. $x^2 + 2x + 1 = 0$

12. $4x^2 = 36$

13. $4x^2 + 4x + 1 = 9$

Solve these equations without the blocks.

14. $4x^2 - 4x + 1 = -9$

15. $x^2 - 10x + 25 = 16$

16. $x^2 + 6x + 9 = 4x^2 - 4x + 1$

17. $-x^2 - 6x - 9 = -25$ (Hint: If quantities are equal, their opposites must be equal.)

18. Explain why some problems had one, or no solution.

COMPARING METHODS

19. Summary

a. Compare the graphical method and the Lab Gear method for the solution of an equal-squares equation. Use examples that can be solved by both methods, and have 0, 1, and 2 solutions.

b. What is the meaning of the *x*-intercept in the graphical method? Where does that number appear in the Lab Gear method?

c. What is the meaning of the *x*- and *y*-coordinates of the intersections of the line and parabola in the graphical method? Where do these numbers appear in the Lab Gear method?

20. Create an equal-squares equation that has two solutions that are not whole numbers. Solve it.

REVIEW FACTORING PRACTICE

Factor these polynomials. One is difficult, one is impossible. The Lab Gear may help for some of the problems.

21. $xy + 6y + y^2$ **22.** $y^2 - 16$

23. $3x^2 + 13x - 10$ **24.** $4x^2 + 8x + 4$

25. $2x^2 + 2x + 1$ **26.** $y^2 - 5y + 6$

27. $y^2 - 4y + 4$ **28.** $x^2 + 8x + 12$

REVIEW MULTIPLICATION PRACTICE

You can multiply polynomials without the Lab Gear and without a table. Picture the table in your mind, and make sure you fill all its spaces. For example, to multiply

$$(2 - x)(7 - 3x + 5y)$$

you would need a 2-by-3 table. To fill the six cells of the table, you would multiply the 2 by 7, by -3*x*, and by 5*y*. Then you would multiply the -*x* by 7, by -3*x*, and by 5*y*. Finally, you would combine like terms. While you think of the six cells of the table, what you actually write on paper looks like this.

$$(2 - x)(7 - 3x + 5y)$$
$$= 14 - 6x + 10y - 7x + 3x^2 - 5xy$$
$$= 14 - 13x + 10y + 3x^2 - 5xy$$

29. Look at the example above, and make sure you understand where each term came from.

Multiply these polynomials without using a table. Combine like terms.

30. a. $(2x - y)(y + 3x)$
b. $(x - 5y)(3x + 2y)$
c. $(ac - b)(2b + 2ac)$
d. $(ab - c)(b - c^2)$

31. a. $(2x - y + 4)(5 - y)$
b. $(2x^2 - y + 4)(5y - x^2)$
c. $(2x - y^2 + 4)(5y - x^2)$
d. $(a + b + c)(2a + 3b + 4c)$