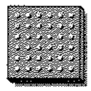
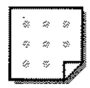
LESSON 9.2

The Pythagorean Theorem

You will need:

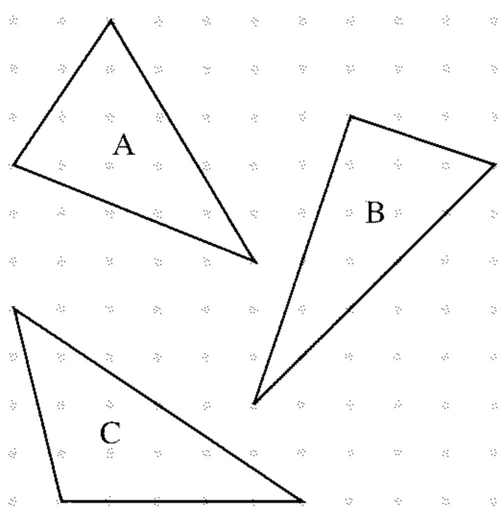
geoboards 

dot paper 

RIGHT TRIANGLES

The corner of a piece of paper can be used to measure a *right angle.*

1. The figure shows three triangles, having a total of nine angles. To do the following problems, you may copy the figure onto your geoboard.
   a. Give the coordinates of the vertex of the right angle.
   b. Give the coordinates of the vertex of the angle that is greater than a right angle.

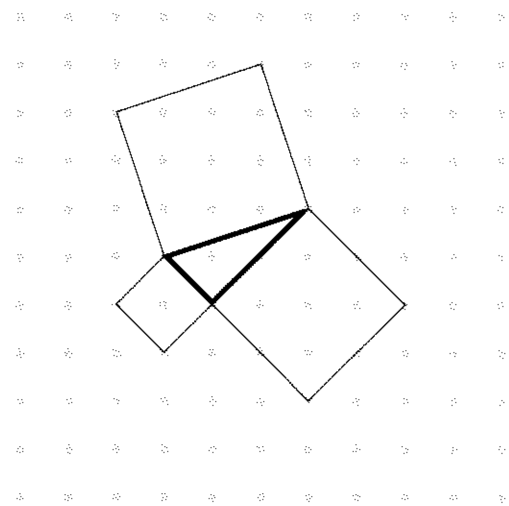


Definitions: An angle that is greater than a right angle is called *obtuse.* An angle that is less than a right angle is called *acute.* A triangle that contains an obtuse angle is called an *obtuse triangle.* A triangle that contains three acute angles is called an *acute triangle.* A triangle that contains a right angle is called a *right triangle.*

2. Which triangle in the preceding figure is acute? Right? Obtuse?

Definition: The two sides forming the right angle in a right triangle are called the *legs.*

The figure shows a right triangle. Three squares have been drawn, one on each of the sides of the triangle.



3. What is the area of each of the three squares?

4. **Exploration** Working with other students, make eight figures like the one above (on geoboards or dot paper). Each figure must be based on a different right triangle. For each one, find the areas of the three squares. Fill out a table like the one below. Study the table for any pattern.

Areas of:

Square on short leg	Square on long leg	Square on hypotenuse
...	...	...

The pattern you probably discovered is called the Pythagorean theorem, after the ancient Greek mathematician Pythagoras. The pattern is about the relationship of the squares on the legs to the square on the hypotenuse for a right triangle.

5. Make an acute triangle on a geoboard or dot paper. Draw a square on each of its sides. Is the sum of the areas of the two smaller squares equal to, greater than, or less than the area of the large square?

6. Repeat problem 5 with an obtuse triangle.

7. **Summary** Explain the following equation.

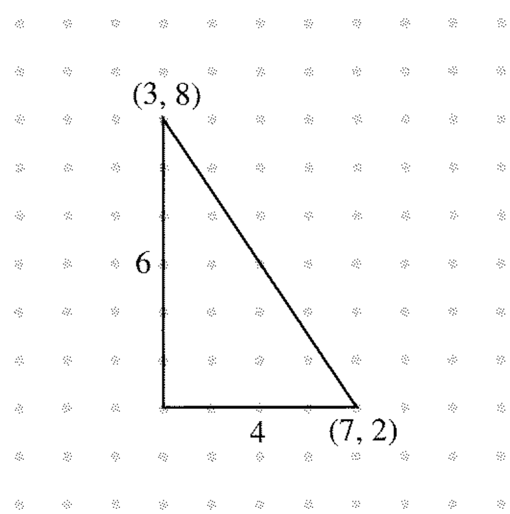
$$\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$$

(Is it true of any triangle? For those triangles for which it is true, what does it mean?)

FINDING DISTANCES FROM COORDINATES

The Pythagorean theorem provides us with a way to find distances in the Cartesian plane. (*Distance* usually refers to Euclidean distance.)

Example: What is the distance between (3, 8) and (7, 2)? First sketch the points.



You can see on the sketch that the length of the legs is 4 for the horizontal leg, and 6 for the vertical leg. Since the triangle shown is a right triangle, we can use the Pythagorean theorem, ($\text{hyp}^2 = \text{leg}^2 + \text{leg}^2$). In this case,

$$\text{distance}^2 = 4^2 + 6^2 = 16 + 36 = 52$$

$$\text{so, distance} = \sqrt{52} = 7.21\ldots$$

8. For the example, if you did not sketch the figure, how could you find the lengths of the legs directly from the coordinates?

Note that the lengths of the legs have been called the *rise* and the *run* when discussing slope. However keep in mind that rise, run, and slope can be positive, negative, or zero, while distances cannot be negative.

9. Consider the two points (3, -4) and (4, -9). Use a sketch if you need to.
   a. Find the rise between them.
   b. Find the run between them.
   c. Find the slope of the line that joins them.
   d. Find the taxicab distance between them.
   e. Find the Euclidean distance between them.

10. Use any method to find the (Euclidean) distance between:
   a. (-1, 2) and (-1, -7);
   b. (-1, 2) and (5, 2);
   c. (-1, 2) and (5, -7);
   d. (-1, 2) and (-1, 2).

11. For which part of problem 10 is the Pythagorean theorem helpful? Explain.

12. Find the distances between:
   a. (8, 0) and (0, -8);
   b. (-8, 0) and (3, -8);
   c. (1.2, 3.4) and (-5.6, 7.89).

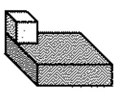
AN OLD PROBLEM

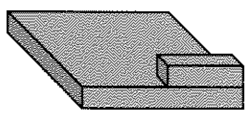
The mathematician Leonardo of Pisa, also known as Fibonacci, posed this problem in 1202.

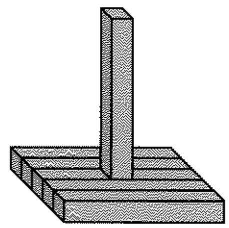
13. Two towers of height 30 paces and 40 paces are 50 paces apart. Between them, at ground level, is a fountain towards which two birds fly from the tops of the towers. They fly at the same rate, and they leave and arrive at the same time. What are the horizontal distances from the fountain to each tower?

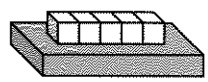
REVIEW *MORE SURFACE AREA*

Imagine these buildings are made by gluing Lab Gear blocks together. The surface area is the total area of all the exposed faces, even the bottom of the building. Find the surface areas.

14. 

15. 

16. 

17. 

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