LESSON 9.8

The Exponent 1/2

THE HALFWAY GROWTH FACTOR

1. A bacterial population is growing exponentially. It is multiplied by nine every day.
   a. Copy and complete the table of the population at half-day intervals.

Time	Population
0	100
0.5	—
1	900
1.5	—
2	—

   b. Write an equation giving the population as a function of time (measured in days).

2. Repeat problem 1 for a population that is multiplied by 25 every day.

3. For problems 1 and 2:
   a. By how much was the population multiplied in half a day?
   b. How are these numbers related to the equation?

4. A tumor that is growing exponentially triples in ten years. By how much is it multiplied in five years?

5. **Generalization** An exponentially growing tumor is multiplied in size by B every ten years and by H every five years. How are B and H related? Explain.

A FRACTIONAL EXPONENT

6. Find x.
   a. $2^5 \cdot 2^5 = 2^x$
   b. $2^3 \cdot 2^3 = x^6$
   c. $(2^4)^2 = 2^x$

7. Find x.
   a. $9^x \cdot 9^3 = 9^6$
   b. $9^x \cdot 9^x = 9^2$
   c. $9^x \cdot 9^x = 9^1$
   d. $B^x \cdot B^x = B^1$

8. Find x.
   a. $(9^x)^2 = 9^6$ b. $(9^x)^2 = 9^1$
   c. $(B^x)^2 = B^6$ d. $(B^x)^2 = B^1$

9. Problems 6-8 suggest a meaning for the exponent 1/2. Explain it.

10. Using this meaning of the exponent 1/2, find the following. (Avoid using a calculator if you can.)
    a. $16^{\frac{1}{2}}$ b. $400^{\frac{1}{2}}$
    c. $25^{-\frac{1}{2}}$ d. $2^{\frac{1}{2}}$

11. Does it make sense to use the exponent 1/2 in the equations you found in problems 1 and 2? Explain your answer.

12. A colony of bacteria was growing exponentially. It weighed 6 grams at noon and 15 grams at 8 P.M. How much did it weigh at 4 P.M.? Explain.

LAWS OF EXPONENTS AND RADICAL RULES

Rules for operations with radicals can be derived from laws of exponents using the fact that

$$x^{\frac{1}{2}} = \sqrt{x}.$$

The following rules assume a and b are nonnegative.

Exponent Rule	Radical Rule
$a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = a^1$	$\sqrt{a}\,\sqrt{a} = a$
$a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} = (ab)^{\frac{1}{2}}$	$\sqrt{a}\,\sqrt{b} = \sqrt{ab}$
$\frac{a^1}{a^{\frac{1}{2}}} = a^{\frac{1}{2}}$	$\frac{a}{\sqrt{a}} = \sqrt{a}$
$\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \left(\frac{a}{b}\right)^{\frac{1}{2}}$	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

13. Check all the radical rules by using $a = 16$ and $b = 9$.

The last rule is especially useful for simplifying rational expressions involving radicals. To be in simple radical form, an expression cannot have any radicals in the denominator or fractions under the radical sign.

Examples:

$$\frac{\sqrt{16}}{\sqrt{8}} = \sqrt{\frac{16}{8}} = \sqrt{2}$$

$$\sqrt{\frac{144}{169}} = \frac{\sqrt{144}}{\sqrt{169}} = \frac{12}{13}$$

$$\frac{\sqrt{48}}{\sqrt{32}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

14. Write problems 14-15 in simple radical form. You can check the answers on your calculator.

a. $\frac{\sqrt{60}}{\sqrt{30}}$ b. $\frac{\sqrt{450}}{\sqrt{18}}$

c. $\frac{\sqrt{18}}{\sqrt{2}}$ d. $\frac{\sqrt{20}}{\sqrt{5}}$

15. a. $\sqrt{\frac{25}{125}}$ b. $\sqrt{\frac{32}{48}}$

c. $\sqrt{\frac{3}{75}}$ d. $\sqrt{\frac{1}{12}}$

SQUARE ROOTS OF POWERS

16. **Exploration** Use your calculator to make a list of the square roots of the powers of ten, from $\sqrt{10^1}$ to $\sqrt{10^{10}}$. Explain any pattern you discover.

17. Explain the pattern you found in problem 16 by using a law of exponents and the exponent 1/2. (Hint: It is not one of the laws listed before problem 13.)

18. Write in simple radical form.

a. $\sqrt{9(10^8)}$ b. $\sqrt{4(10^7)}$

c. $\sqrt{3(10^6)}$ d. $\sqrt{2(10^5)}$

CHALLENGE *ESTIMATING POPULATION*

19. The population of California was 3,426,861 in 1920 and 15,717,204 in 1960. Assume it grew exponentially and estimate the population in:

a. 1940; b. 1949.