LESSON 9.10

Blowups

You will need:

graph paper

BIGGER BOXES

The Real Bag Company makes cardboard boxes. One of the boxes is called the Banker's Box. It has the dimensions: length, 16 in.; width, 12 in.; height, 10 in. Another box, the Square Pak box, has the dimensions: length, 12 in.; width, 12 in.; height, 10 in. Sid, a Real Bag Box Division Manager, decides that new boxes need to be manufactured, the Caterer's Crate and the Great Pak.

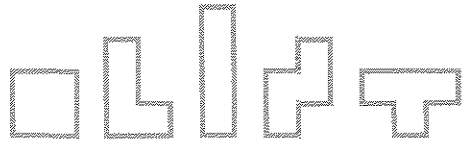
1. **Exploration**

a. The Caterer's Crate will have two dimensions the same as the Banker's Box, and the third dimension multiplied by two. Sid asks his colleague Li Ann whether the volume of the box would be increased the most by multiplying the length, the width, or the height by two. What should she answer? Explain.

b. The Great Pak will have a square base and a volume that is double that of the volume of the Square Pak. Sid asks his colleague Annette (who owns a calculator) to find three choices for the dimensions of the new box. What should she answer? Explain.

STRETCHING POLYOMINOES

Sid, Annette, and Li Ann like to spend their lunch breaks working geometric puzzles. (They should have become math teachers.) Here is a puzzle they have been working on, using the tetrominoes.



square l i n t

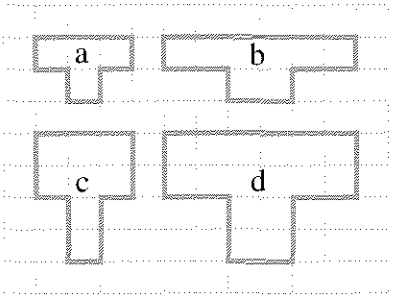
For each of the tetrominoes, they created three new polyominoes. The first one by doubling all horizontal dimensions, the second one by doubling all vertical dimensions, and the third one by doubling both horizontal and vertical dimensions. For example, the t tetromino led to the creation of three new polyominoes.

a. original

b. doubled horizontally

c. doubled vertically

d. doubled both ways



2. **Exploration** Draw all 15 *stretched* tetrominoes. For each one, find its area and perimeter. Keep your work clearly organized, so you can find a pattern to the areas and perimeters. (The area pattern is the easier of the two.) You will need to refer to this data to do the problems in the next two sections.

PERIMETER

Call the perimeter of a tetromino p. It is made up of some horizontal segments and some vertical segments.

Let h = total length of the horizontal segments.
Let v = total length of the vertical segments.

3. Express p in terms of h and v.

4. a. Find h and v for the t tetromino.
b. Show that the perimeter of the vertically stretched t tetromino is $h + 2v$.
c. What is the perimeter of the horizontally stretched t tetromino in terms of h and v?
d. What is the perimeter of the horizontally and vertically stretched t tetromino in terms of h and v?

5. In problem 4 you found formulas that related the perimeters of the three stretched t tetrominoes to the perimeter of the original t tetromino. Explain why these formulas work for all the tetrominoes.

6. What is the sum of the perimeters of the two polyominoes that were stretched in only one dimension? Use factoring to see how this sum is related to the original perimeter.

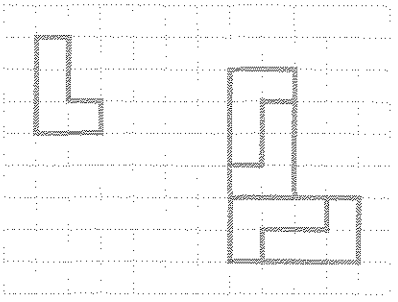
7. **Generalization**
a. Repeat the perimeter investigation, but stretch the tetrominoes by tripling dimensions. You do not need to draw the tripled tetrominoes, just use algebra. Find a formula relating the perimeters of the tripled tetrominoes to h, v, and p for the original tetromino.
b. Repeat this investigation, but this time stretch by a factor of n.

AREA

8. Refer to your data on the area of the 15 stretched (doubled) tetrominoes, and experiment with other polyominoes. If the original area of a polyomino is A, what is the area of the polyomino stretched by doubling
a. horizontally?
b. vertically?
c. both horizontally and vertically ?

9. a. Draw the l and t tetrominoes, with both their horizontal and vertical dimensions doubled.
b. Repeat part (a), tripling the dimensions instead of doubling.

10. **PUZZLE** Tile the blown-up tetrominoes you drew with copies of the original l and/or t tetrominoes. Example:



11. How many tetromino tiles did you need to cover the blown-up tetrominoes? How is this related to the area of the blown-up tetrominoes?

12. a. Draw a pentomino.
b. Draw a copy of it, with horizontal and vertical dimensions multiplied by two.
c. Repeat with the original dimensions multiplied by three.
d. Repeat with the original dimensions multiplied by four.

13. Predict the area of each figure you drew in problem 12. Check your predictions.

14. **Generalization** When both horizontal and vertical dimensions are multiplied by k, by what is the area multiplied? Explain.

BACK TO WORK

After their lunch break, Sid, Li Ann, and Annette had to attend to more box problems.

15. They created a new box by multiplying all the dimensions of the Banker's Box by two. Make a sketch of the original box and the new box. What would the volume of the new box be? How many times greater is this than the volume of the Banker's Box?

16. If they created a new box by multiplying all the dimensions of the Square Pak by three, what would its volume be? How many times greater is this than the volume of the Square Pak?

17. **Generalization** When all the dimensions are multiplied by k, by what is the volume multiplied?

18. What are the dimensions of a box that is a perfect cube and has the same volume as the Square Pak? Explain.

19. What are the dimensions of a box that is a perfect cube and has double the volume of the Square Pak? Explain.

REVIEW *SCIENTIFIC NOTATION*

20. In June of 1990 the national debt of the United States was \$3.1 trillion. The population of the U.S. at the same time was about 250 million. Therefore, the debt per person was

$$\frac{3.1 \text{ trillion}}{250 \text{ million}}.$$

a. Express both of these numbers in scientific notation.

b. What was the debt per person? Express your answer in ordinary decimal notation and in scientific notation.

REVIEW *WHAT'S YOUR SIGN?*

Do not use a calculator for these problems.

21. Is x positive or negative, or is it impossible to know? Explain.

a. $(-2)^x = -524{,}288$

b. $2^x = 1/131{,}072$

c. $(-2)^x = 262{,}144$

d. $x^{11} = -177{,}147$

e. $x^{12} = 531{,}441$

f. $x^{13} = 1/1{,}594{,}323$

REVIEW THE CHESSBOARD

According to an old legend, a King decided to reward the inventor of the game of chess. "I am immensely rich. Whatever you ask for will be yours." The inventor replied, "All I ask is for one cent on the first square of the chessboard; two cents on the next square; four cents on the next square; and so on, doubling the amount each time, until the last square on the chessboard." (The legend actually specifies grains of rice, not cents.)

22. Find out how many cents the King owed the inventor. Express the final answer two ways: in terms of a power of two, in cents; and as a number of dollars, in scientific notation.

23. **Project** Is the money paid the inventor as much as the budget of:

a. a toy store?

b. a multi-national corporation?

c. the State of New York?

d. the United States?

DISCOVERY DECIMAL EXPONENTS

24. Use decimal exponents (to the nearest hundredth) to approximate 100 as a power of:

a. 2 b. 3 c. 4

d. 8 e. 9 f. 10

REVIEW EQUAL RATIOS

Solve for N.

25. $\frac{3N-2}{5} = \frac{N+2}{2}$

26. $\frac{3N-2}{15} = \frac{N+2}{6}$

Solve for x. If you cannot find an exact value, approximate to nearest thousandth.

27. $\frac{x}{8} = \frac{3}{4}$

28. $\frac{4}{10} = \frac{400}{x}$

29. $\frac{1}{x} = \frac{x}{2}$

REVIEW DISTRIBUTIVE LAW PRACTICE

Find these products.

30. $2y(2x - y + 6)$

31. $3x(2x - 3)$

32. $(y - 4)(y + 3)$