LESSON 10.8

Lines Through Points

You will need:
- graph paper
- graphing calculator (optional)

1. **Exploration** The linear equation $y = x - 1$ has (2, 1) as a solution. Make up several more linear equations in x and y that have (2, 1) as a solution. Compare your solutions with those of other students. How many different linear equations have this solution?

FINDING COORDINATES

Hint: The problems in this section and the following one can be solved by graphing carefully.

2. A line having slope -2 passes through the point (-4, 3). Give the coordinates of three more points on the line.
3. A line having slope -3 passes through the point (5, 12). The points (a, 5) and (0, b) are on the same line. Find a and b.
4. A line passes through (2, 1) and (-2, -1). Give the coordinates of three more points on the line.
5. The points (7, -2) and (6, 2) are on a line. The points (a, 5) and (0, b) are on the same line. Find a and b.

LINES THROUGH A POINT

6. Which of the following lines pass through the point (1, -1)?
   a. $5x - 5y = 10$ b. $5x + 5y = 10$
   c. $2x - 3y = 6$ d. $-3x + 2y = 6$
7. The line $y = mx - 1$ passes through the point (3, 2). What is m?
8. The line $y = (-1/3)x + b$ passes through the point (3, 2). What is b?

FINDING THE EQUATION OF A LINE

9. Graph the line that passes through the points (1, 3) and (3, 8). Find its equation.

Ellen and Sandor wanted to find the equation of a line passing through (4, 5) and (8, -3) *without using graphing*.

10. Ellen could tell by imagining the graph that the slope of the line must be negative and the y-intercept must be greater than 5. Explain.

Ellen knew that the equation could be written in slope-intercept form as $y = mx + b$. "All I have to do is find m and b," she thought. Using the point (4, 5), she substituted values for x and y and wrote this equation in m and b,

$$5 = m(4) + b$$

which she rewrote as $5 = 4m + b$.

11. What equation in m and b did she write, using the point (8, -3)?

12. a. Find the values of m and b that satisfy both of Ellen's equations.

b. Write the slope-intercept equation of the line passing through the points.

Sandor knew that the equation could be written in standard form as $Ax + By = C$. He substituted values for x and y and wrote two equations. One was $A(4) + B(5) = C$, which he rewrote as $4A + 5B = C$.

13. a. What was the other equation?

b. Find some values of A, B, and C that satisfy both equations. (Many solutions are possible.)

c. Write in standard form an equation of the line passing through the points.

d. Compare your answer to (c) with other students' answers.

14. Show that Ellen and Sandor got equivalent answers, one in slope-intercept form and the other in standard form.

15. Find the equation of a line having slope 1.5 that passes through the point (0.5, 4).

16. Find the equation of the line through the points (2.3, 4.5) and (-6, -7). (You may round off the parameters.)

17. Summary Explain, with examples, your strategies for finding the equation of a line,

a. when you know its slope and the coordinates of a point on it;

b. when you know the coordinates of two points on it.

CELSIUS-FAHRENHEIT CONVERSION

Water freezes at 0° Celsius, which is 32° Fahrenheit. Water boils at 100° Celsius, which is 212° Fahrenheit.

18. A temperature reading can be converted from Fahrenheit to Celsius by using the formula $C = mF + b$. Find m and b by using the fact that $C = 0$ when $F = 32$, and $C = 100$ when $F = 212$.

19. Find a formula for converting Celsius to Fahrenheit.

20. What is the relationship between the formulas that you found in problems 18-19?

21. When the temperature increases by n degrees on the Celsius scale, by how much does it increase on the Fahrenheit scale? Explain.

ADDING POINTS

A line passes through the points (2, 4) and (6, 8). If you add the x-coordinates and the y-coordinates of these points you get the point (8, 12). Call this point the *sum* of the points.

22. What point is the *difference* of the points?

23. a. Find the equation of the line through (2, 4) and (6, 8).
b. Does this line also pass through the sum and the difference of (2, 4) and (6, 8)?

24. Write the equation of any line and find the coordinates of two points on the line. Find their sum and difference. Does the line pass through the sum and difference points?

25. Find the equation of a line such that the sum and the difference of any two points on the line is also on the line. To find this line, it may help to experiment with graphs. Compare your answers to problems 23-24 with other students' answers.

26. Summary What kinds of lines contain the sum and the difference of any two points on the line? Explain, giving examples and counter-examples.

27. What's wrong with this reasoning? (Hint: Think about problems 18-26.)

0°C = 32°F
100°C = 212°F
Adding equals to equals:
100°C = 244°F

DISCOVERY ***REAL WORD PROBLEM***

28. Rearrange the letters in the sentence

I'm a pencil dot.

to create an appropriate mathematical two-word phrase. (Hint: The second word has five letters.)