LESSON 12.4

Mathematical Models in Science

You will need:

graph paper

materials for experiments (See below)

HEATING AND COOLING GASES

Doing science often means finding mathematical models that fit experimental data. In 1787 the French scientist Jacques Charles discovered that when a gas is kept at constant pressure, it expands when heated and contracts when cooled. For gases under constant pressure, *volume is a linear function of temperature*.

At a certain pressure a gas has a volume of 500 cubic centimeters at 27°C. Kept at the same pressure, it expands to 605cc at 90°C.

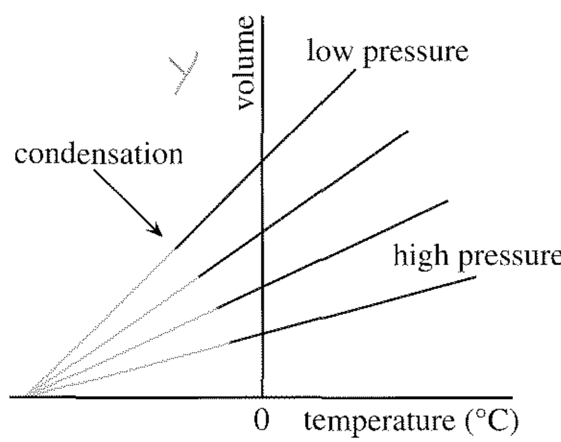
1. Find an equation that gives the volume (V) of this gas as a linear function of the temperature (T).
2. Find the volume of the gas at 0°C.
3. When kept at this pressure, how much does this gas expand for every 1°C increase in temperature?

When they get cold enough, gases condense (turn into liquids). If they did not, the temperature for which the volume would be 0 is called *absolute zero*, the lowest possible temperature.

4. Use the equation you wrote in problem 1 to figure out what temperature absolute zero must be.

This graph shows how the volume of a certain gas varies with temperature, when kept at constant pressure. Each line represents a different pressure. The point where the red line ends and the blue line starts is the condensation point. Only the red lines represent actual data.

Charles's Law

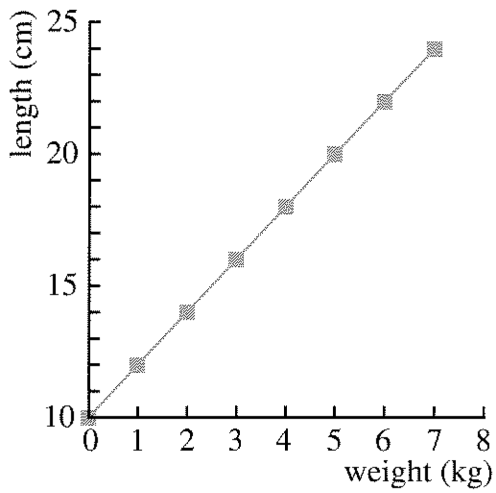


Summary

5. As the pressure increases, what happens to the slopes of the lines? What does this mean in terms of the application?
6. What is the meaning of the y-intercept of the lines? As the pressure increases, how does it change?
7. How does the condensation point vary with pressure?
8. Why do all the blue lines intersect at one point? What is the point's significance?

STRETCHING IT

The length of a spring is related to the weight that hangs from it. The following figure shows a graph from an experiment with a certain spring.



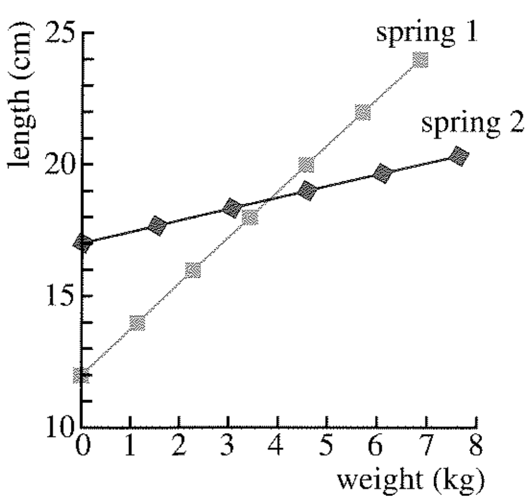
9. a. What was the length of the spring before any weight was added?

b. How many centimeters did the spring stretch for each kilogram of weight?

c. What is the equation that relates length to weight?

10. Can the graph be indefinitely extended to the right? Explain.

11. This graph shows data for two other springs. Which spring is stiffer? Which one is longer? Explain.



DEPENDENCE AND INDEPENDENCE

Paul, a forest lover, knew how to estimate the temperature from listening to cricket chirps. His grandmother had taught him to count the number of chirps per minute, divide by 4, and add 40 to get the temperature in Fahrenheit.

12. Write an equation for the temperature (T) as a function of the number of chirps (C).

13. What would Paul estimate the temperature to be if he counted 180 chirps per minute?

14. According to the model, at what temperature would the crickets cease to chirp?

Since the number of chirps depends on the temperature, and not vice-versa, we call number of chirps the *dependent* variable and temperature the *independent* variable.

In algebra we usually call the independent variable x and use the horizontal axis for it. We call the dependent variable y and use the vertical axis for it. Likewise, we often express the relationship between the two variables by writing the dependent variable as a function of the independent variable.

15. Write an equation for the dependent variable (number of chirps) as a function of the independent variable (temperature). Hint: Use your equation from problem 12 and solve for C in terms of T.

In an experiment the independent variable is the variable we change or manipulate. Then we observe and record the effect on the dependent variable.

16. Which variable is dependent and which is independent in problem 9? Explain.

YOUR OWN EXPERIMENTS

For each experiment, problems 17-20, do the following:

a. Discuss the relationship you expect between the two variables.
b. Identify the dependent and independent variables.
c. Carry out the experiment and collect the data in a table.
d. Make a graph.
e. Interpret the graph.
f. If possible, write an equation relating the variables.
g. Draw some conclusions.

17. *Spring:* The length of a spring as a function of the weight that hangs from it — **You will need** a spring and several identical weights. Start by letting the spring hang freely. Measure its length. Then add the weights one by one, each time measuring the length of the spring as it stretches.

18. *Fall:* The time it takes for Lab Gear blocks to fall as a function of the number of blocks — **You will need** a stopwatch and 20 or more x^2-blocks. Line up x^2-blocks so that if the first one is pushed, each block will knock down the next block in succession.

19. *Summer:* The time it takes to do "the wave" as a function of the number of people involved. **You will need** a stopwatch. Decide on an order for the wave. Appoint a student (or the teacher) to be the timer. When the timer says "Go," take turns getting up and sitting down. Repeat the experiment for different numbers of people.

20. *Winter:* The height of an ice column as a function of the height of the corresponding water column — **You will need** some drinking straws, chewing gum, ice (or access to a refrigerator). Plug the bottom of a straw with gum. Fill it to a certain height with water. Mark and measure the height of the water column. Do it again with different amounts of water in other straws. Freeze them. Mark and measure the height of the column of ice.

21. Report Write an illustrated lab report on an experiment you conducted. This can be one of the ones presented in this section or another one of your own design. Include the data you collected, a graph, and an equation, if you found one. Describe the conditions in which you conducted the experiment, your expectations, and your conclusions.