LESSON 13.2

Advanced Penmanship

You will need:

graph paper

PEN PARTITIONS

Assume that you have 50 feet of fencing to build a rectangular pen. You plan to use the garage wall as one side of the pen, which means you need to use your fencing for only three of the four sides. Since you are considering adopting more pets, you want to investigate what happens when you use some of the fencing to divide the pen into two or more parts by building partitions inside the pen, at a right angle to the wall.

1. Make a rough sketch of what this pen might look like,
   a. having no internal partitions;
   b. divided into two sections.

2. With no partitions, is it possible to get a square pen? If so, what are its dimensions?

3. With one partition, is it possible to get two square sections? If so, what are their dimensions?

Call the side of the pen parallel to the wall the *length,* and the distance between the wall and the side opposite the wall x.

4. Imagine you are dividing the pen into two parts. Make a table having three columns: x, the length, and the total area of the pen.

Generalizations

5. Look for patterns in your table. Express algebraically as functions of x,
   a. the length;   b. the area.

6. What is the equation that expresses the length as a function of x, if the pen is divided into the given number of parts. (Make sketches. If you need to, make tables like those in problem 4.)
   a. 1   b. 3
   c. 4   d. n

7. Repeat problem 6, but this time find the area as a function of x.

GRAPHS OF AREA FUNCTIONS

This section is about the graphs of functions like the ones you found in problem 7.

8. Using graph paper, graph each of the following functions. To make comparison easier, use the same graph, or at least the same scale, for all your graphs. To see all four graphs clearly, use a scale that will show values from $^-5$ to 15 for x and from $^-50$ to 50 for y. When making a table of values, use both negative and positive values for x. Keep these graphs, because you will need them in the next section.
   a. $y = x(12 - x)$
   b. $y = x(12 - 2x)$
   c. $y = x(12 - 3x)$
   d. $y = x(12 - 4x)$

9. For each graph,
   a. label the graph with its equation;
   b. label the x-intercepts;
   c. label the vertex.

10. Write a brief description comparing the four graphs. Describe how the graphs are the same and how they are different.

11. Without graphing, guess the vertex on the graph of $y = x(12 - 6x)$. Explain how you arrived at your guess.

DIFFERENT FORMS

As you learned in Lesson 1 the equations of parabolas through the origin can be written in the form $y = ax(x - q)$.

12. For each parabola described in (a-d), find a function of the form $y = ax(x - q)$:

a. x-intercepts: 0 and 12, vertex: (6, 36)
b. x-intercepts: 0 and 6, vertex: (3, 18)
c. x-intercepts: 0 and 4, vertex: (2, 12)
d. x-intercepts: 0 and 3, vertex: (1.5, 9)

13. How are the intercepts and the vertex related to the values of a and q in the equation $y = ax(x - q)$?

The equations in problems 8 and 12 have the same graphs. You can verify this by checking that they have the same vertices and intercepts, and in fact that for any x they yield the same y. In other words, the equations are equivalent. We can use the distributive law to confirm this. For example, for problem 8a:

$$y = x(12 - x) = 12x - x^2$$

And for problem 12a:

$$y = -x(x - 12) = -x^2 + 12x$$

14. Show that the other three pairs of equations in problems 8b-d and 12b-d are equivalent.

It is possible to convert equations like the ones in problem 8 to the form $y = ax(x - q)$ by factoring. For example:

$$x(24 - 6x) = 6x(4 - x) = -6x(x - 4)$$

15. Fill in the blanks:

a. $x(24 - 2x) = 2x($______$)$
b. $x(24 - 3x) = -3x($______$)$
c. $x(24 - 4x) =$ ___$(x - 6)$

16. Write in the form $y = ax(x - q)$ and find the vertex and the intercepts.

a. $y = x(12 - 6x)$
b. $y = x(50 - 5x)$
c. $y = x(50 - 3x)$
d. $y = x(50 - (n + 1)x)$

MAXIMIZING AREA

17. If you have to use part of the 50 feet of fencing for a partition to divide the pen into two equal parts, what is the largest total area you can get for the enclosure? Explain how you got your answer, including a sketch and graph if necessary.

18. Solve problem 17 if you want to divide the pen into three equal parts.

19. Solve problem 17 if you want to divide the pen into n equal parts.

20. Look at your solutions for problems 17, 18, and 19. In each case look at the shapes of the subdivisions of the pen having the largest area. Are they always squares? Are they ever squares? Does the answer to this depend on the value of n? Explain.

21. Look at your solutions for problems 17, 18, and 19. In each case look at how much of the fencing was used to construct the side parallel to the garage for the pen having maximum area. What fraction of the fencing was used to construct this side? Does the answer depend on the value of n? Explain.

22. **Report** Imagine you are the representative of a fencing company presenting information to a customer. Write a complete illustrated report, making clear who the customer is and what the pens are needed for. Explain how to maximize the area of the pens for a given amount of fencing. Discuss both divided and undivided pens.