LESSON 13.5

Packing and Mailing

You will need:

graph paper

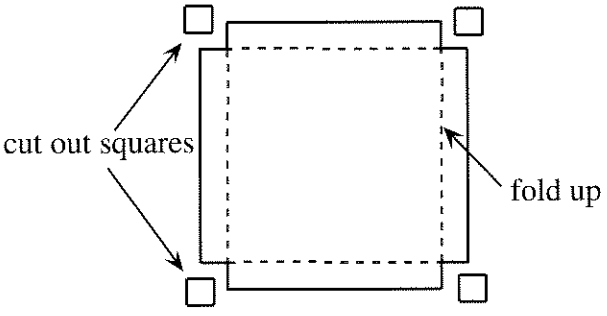
centimeter grid paper

scissors

tape

MAXIMIZING VOLUME

You can make cardboard trays to hold 1-cm^3 cubes. Start with an 18-cm-by-18-cm piece of grid paper. Cut a square out of each corner and fold up the sides to form a tray.



1. **Exploration** Work with other students to make as many different trays as you can by cutting square corners out of an 18-cm-by-18-cm piece of paper or cardboard. Figure out which tray holds the most cubes.

2. Make a table showing the side of the square corner that was cut out, the area of the base, and the number of cubes the tray would hold. (For example, if a 2-by-2 square is cut out at each corner, the area of the base should be 196 cm^2, and the tray should hold 392 cubes.)

3. If the side of the square cut out of the corner is x,
   a. what is the area of the base?
   b. what is the volume of the tray?

4. Make a graph of the volume of the tray as a function of x. Include some fractional values of x.

5. What is the height of the tray that will give the maximum volume?

6. What are the x-intercepts of the graph? Interpret them in terms of this problem.

7. Draw a vertical line through the highest point on the graph. Are the x-intercepts equidistant from it?

8. Extend the graph in both directions by using a few more values for x beyond the x-intercepts.

9. Explain why the points you added in problem 8 do not represent the tray problem.

10. Is the graph a parabola? Explain, giving as many reasons as you can for your answer.

11. **Generalization** Find the height which would give the maximum volume if the initial piece of paper had the following dimensions. You may want to use tables of values.
    a. 12 by 12 b. S by S

STORING CUBES

Suppose you want to make a cardboard tray for storing 100 centimeter cubes. The base does not have to be square.

12. **Exploration** What should the dimensions of the tray be so it will contain the least cardboard? Explain.

13. Repeat problem 12 for:

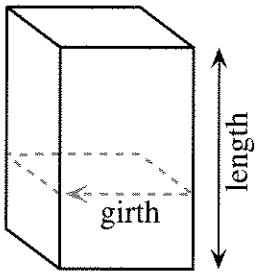
a. 50 cubes; b. 200 cubes;

c. 500 cubes; d. 1000 cubes.

14. **Project** Write an illustrated report explaining a strategy for solving this problem for N cubes.

POSTAL REGULATIONS

The U.S. Postal Service will not mail by Priority Mail™ anything that weighs more than 70 pounds or exceeds 108 inches in combined length and girth. (The girth is the distance around, as shown in the figure.)



15. **Exploration** Find the dimensions for a box that would satisfy the Priority Mail™ requirements and would hold as large a volume as possible.

16. Suppose you want to mail a box full of 20-inch-long dowels. What are the dimensions of the rectangular box having the largest volume that would satisfy postal regulations and would accommodate the dowels?

17. Repeat problem 16, this time for 12-inch-long dowels.

18. **Project** A lumber company needs to pack dowels in boxes that can be sent by Priority Mail™. Boxes need to be designed to ship dowels of each length. Explain, with examples, how to find dimensions for such boxes that will allow the packing of the maximum number of dowels.

19. A shipping company has the following rules:

- maximum length: 108 inches
- maximum length plus girth: 130 inches

In addition, they recommend two inches cushioning on all sides for fragile items. What is the largest volume possible for the contents of the package in the case of fragile items?