LESSON 13.6

Solving with Squares

You will need:

the Lab Gear 

In this chapter you have used quadratic functions to solve problems involving finding a maximum area. In the next chapter you will be faced with problems for which it will be useful to solve quadratic equations. In this lesson we start to prepare for this.

EQUAL SQUARES

In Chapter 7, Lesson 7, you solved quadratic equations using the equal squares method. Some problems are easy to solve this way. For example,

$$x^2 - 10x + 25 = 16$$

can be written

$$(x - 5)^2 = 4^2$$

with a perfect square on each side.

1. Solve this equation. (Remember: There are two solutions.)

It is not necessary for the number on the right to be a perfect square, since you can take the square root of any nonnegative number.

Example:

$$x^2 - 10x + 25 = 7$$
$$(x - 5)^2 = 7$$
$$x - 5 = \sqrt{7} \text{ OR } x - 5 = -\sqrt{7}$$
$$x = 5 + \sqrt{7} \text{ OR } x = 5 - \sqrt{7}$$

Using your calculator, you can find decimal approximations for the two solutions:

$$x \approx 7.646 \text{ or } x \approx 2.354.$$

Solve these equations using the equal squares method. First give exact answers (using radicals if necessary); then find decimal approximations. Not all are possible.

2. $x^2 - 10x + 25 = 8$

3. $y^2 + 6x + 9 = 15$

4. $x^2 - 24x + 144 = 12$

5. $4r^2 - 4r + 1 = 6$

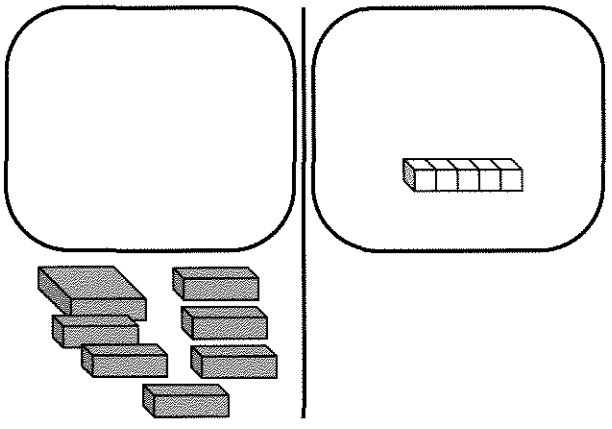
6. $9s^2 + 12s + 4 = 21$

7. $y^2 - 14y - 49 = -20$

COMPLETING THE SQUARE

In this section you will learn how to turn certain quadratic equations into equal squares equations that you know how to solve.

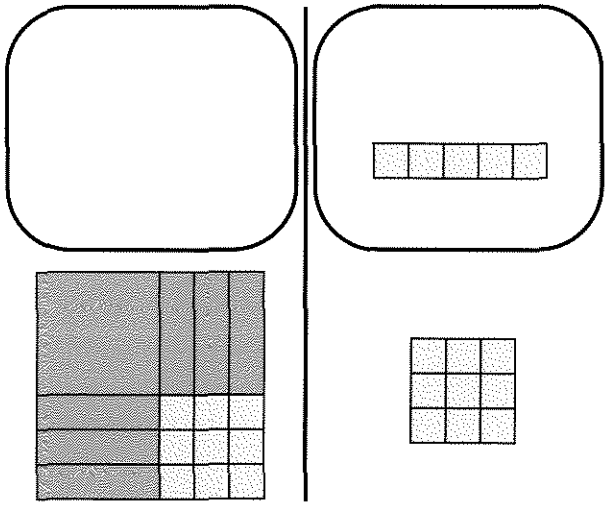
8. Write the equation shown by this figure.



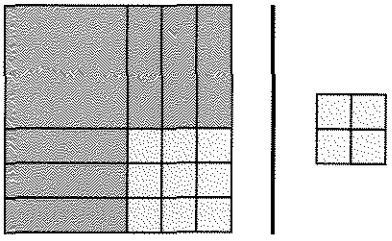
We will add the same quantity to both sides, so that the left side is a perfect square. This is called *completing the square*.

9. a. What number was added to both sides of the figure on the next page to make the left side a perfect square?

b. Write the resulting equation.



10. The right side can be simplified. The resulting equation is shown in the next figure. Write and solve this equation using the equal squares method. You should get two solutions.



Complete the square to solve these equations. You will need to rearrange blocks and add or subtract the same amount on both sides in order to get equal squares.

11. $x^2 + 2x - 3 = 0$ **12.** $x^2 + 12x = -11$

13. $x^2 + 4x = 0$ **14.** $x^2 + 10x - 6 = 5$

15. $x^2 + 8x = 20$ **16.** $x^2 + 6x + 9 = 25$

17. **Generalization** Explain how to figure out what number to add to both sides of an equation of the form $x^2 + bx = k$ in order to get an equal squares equation. Use sketches and examples.

SQUARE PRACTICE

Solve these equations by completing the square. Show all your work. Include a sketch showing the equal squares.

18. $x^2 + 8x = 33$ **19.** $x^2 + 4x = 96$

20. $x^2 + 6x = 55$ **21.** $x^2 + 10x = 56$

Solve these equations. Show your work.

22. $x^2 - 8x = 33$ **23.** $x^2 - 4x = 96$

24. $x^2 - 4x = -96$ **25.** $x^2 + x = 6$

Solve these equations. Show your work. Give exact answers, then find decimal approximations to the nearest hundredth.

26. $x^2 - 8x + 3 = 0$ **27.** $x^2 - 5x - 8 = 0$

28. $x^2 - 4x + 1 = 6$ **29.** $x^2 - 7x - 4 = 0$

QUADRATIC EQUATIONS CHECKPOINT

Solve two of these equations by factoring (and the zero product property), and two by completing the square.

30. $x^2 + 18x = 0$ **31.** $x^2 + 5x = 2.75$

32. $x^2 + 2x - 8 = 0$ **33.** $x^2 + 7x + 12 = 0$

While it is somewhat cumbersome, completing the square is an important technique when dealing with quadratic expressions. Unlike factoring, you can use it to solve any quadratic equation. In addition, we will use completing the square repeatedly to get more understanding of quadratic functions and to develop more efficient ways to solve quadratic equations.

34. Find a quadratic equation having solutions 5 and -2.

35. Find a quadratic equation having solutions $2 + \sqrt{5}$ and $2 - \sqrt{5}$.

36. Multiply. $(x - (4 + \sqrt{3}))(x - (4 - \sqrt{3}))$ (Hint: Carefully remove the inside parentheses and then set up a three-by-three multiplication table.)

37. You should have obtained a quadratic expression in problem 36. Set it equal to zero, and solve the equation.