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Make These Designs

"Make These Designs" is an activity that I have used in algebra classes over many years. It can be done with just about any electronic grapher. Its purpose is to reinforce students' understanding of the connection between the graph of a linear function and the parameters m and b in the formula $y = mx + b$.

Students are given a set of designs (see **fig. 1**) that they must create on their electronic grapher by entering functions of the form $y = mx + b$. The challenge is finding the right values for m and b. Before going on, readers should try to reproduce the graphs. This activity is popular because in many classes, students see it as a break from the usual routine. As students show partial results and ask questions, the teacher has many opportunities to focus their attention on the underlying mathematics. When a question arises that is of concern to enough students, interrupt individual work for a whole-class discussion, using the overhead projector to demonstrate.

This activity is popular

Students need not solve the problems in any particular order. As they work on one picture, they may accidentally create another one. Allowing them to set their own path through the activity makes it more enjoyable and gives them more ownership. In addition, immature competitiveness—"Which one are you on? I'm ahead of you"—becomes difficult, thus freeing the students to concentrate on the mathematics.

It is important that students keep records of how each image was achieved and participate in some small-group or all-class discussion of these records. Without such discussions, students may get through the activity and not learn all they can from it, even if it appears to go well. Some questions that can be asked during these discussions follow:

- How do you make lines steeper? Less steep?
- How do you make lines that go "uphill"? "Downhill"?
- How do you make lines horizontal? Vertical?
- How do you make parallel lines?
- How do you make a parallel to an uphill line to the left of the original? To the right?
- How do you make a parallel to a downhill line to the left of the original? To the right?
- How do you make a parallel to an uphill line higher up? Lower down?
- How do you make a parallel to a downhill line higher up? Lower down?

As students work on the problems, circulate among them, prodding them to improve their designs. On the graph in **figure 1a,** for example, encourage students to fill out the star, since they often fail to include lines that make an angle of less than 45 degrees with the x-axis. For an account of a similar activity and the questions that come up for students and teacher, see Magidson (1992).

Note the questions about left and right. It turns out that if lines are steep, as in **figures 1g** and **1h,** students often see the b parameter as moving the lines left or right, even though teachers see it as moving them up or down. Having the left-right discussion helps students see that changing the value of b has different effects on the x-intercept, depend-

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This department offers a forum where teachers can discuss issues related to technology in the mathematics class. Tips for using technology, classroom activities, and ideas related to teaching and learning mathematics with technology are all appropriate. Ideas that may not fill an entire article are encouraged. We particularly welcome tips involving calculators.

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ing on whether the line is uphill or downhill, whereas it always affects the y-intercept the same way. In fact, use this good opportunity to observe that b *is* the y-intercept. A discussion of how to find the x-intercept through symbol manipulation is a nice detour from this activity. Because this level of discussion is rather sophisticated, do not expect it the first time that students approach this activity. However, clarifying this concept helps in dealing with students' questions about moving lines left and right and gives somewhere to go with the students who quickly master the basic ideas about slope and intercept.

Note also that students are asked to graph vertical lines. It is, of course not possible to do it using the "$y =$" format, and other features of the grapher must come into play. Actually, it is possible, but not easy. How can it be done?

WORKING AT MANY LEVELS

This activity is an example of an approach to curriculum that offers both access and depth in the same lesson. Access, because no one is frozen out of the activity: all students can understand the question, get started, and find a challenge to stretch their own understanding. Depth, because many ways are available to increase the mathematical payoff and to keep even the strongest students challenged. The slogan used in the Logo community to describe such curricula is "No threshold, no ceiling."

Whether a given design is reproduced accurately is largely a matter of opinion. Students can be asked to capture the general look of a given picture or to produce a nearly exact replica. The former can be done by trial and error and helps students develop a feel for the effect of m and b in a general sort of way. The latter requires a very clear understanding of the effect of the parameters, not to mention familiarity with the features of the electronic grapher. The level of competence sought depends on whether this activity is being used early or late in the process of learning about linear functions and their graphs. Different levels of accuracy may be expected from different students, as long as everyone is being challenged to move forward in his or her understanding. Certainly for beginners, using about ten lines is sufficient to show an understanding of the particular figure, even though the figure may include up to sixteen lines.

The graphs are mostly grouped in pairs, each of which tries to make a certain point. The first two graphs (**figs. 1a** and **1b**) show lines with various slopes but the same intercept; however, the intercept changes from the first to the second graph. The next two examples (**figs. 1c** and **1d**) make the point about uphill versus downhill graphs, keeping the slope constant but varying the intercepts. The next two graphs (**figs. 1e** and **1f**) address the special cases of horizontal and vertical lines. The next pair (**figs. 1g** and **1h**) raises the issue of moving lines to the left or right. The next two graphs (**figs. 1i** and **1j**) are an attempt to explore symmetry across the x-axis and y-axis. However, avoid getting heavy-handed in demanding specific understandings right away. The central purpose of the activity is to

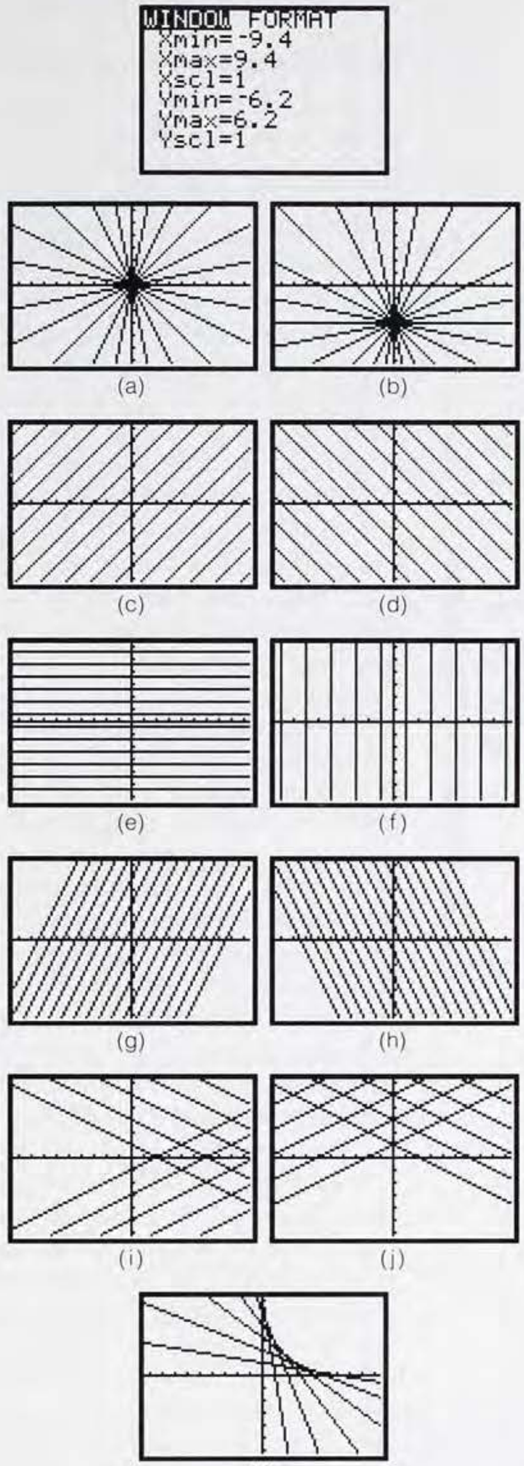

Fig. 1
Make these designs.

advance students' grasp of the role of the parameters m and b. Any other learning is a bonus.

MAKING VERSUS "NOTICING"

Many curriculum materials attempt to get the slope-intercept idea across by way of a lesson in which students are asked to graph several lines from the equations supplied by the curriculum writer. For example, they may be asked to graph $y = x$, $y = 2x$, $y = 3x$, and so on. Then they are asked, "What do you notice?" The process is then repeated for $y = x$, $y = x + 1$, $y = x + 2$, and so on. Surely some students do figure out what is going on in this type of lesson, but many do not. Part of the reason is that the graphing phase of the lesson does not engage the student intellectually; it is just a matter of entering the functions suggested by the worksheet. After looking at the graphs, students often do not notice what teachers want them to notice, and teachers are forced to give sledgehammer hints. In effect, students end up seeing the activity as one in which they have to guess what the teacher is looking for. For an account of what students sometimes "notice" in this context, see Goldenberg (1988, 1991).

Students concentrate on the mathematics

In contrast, Make These Designs forces students to think throughout the activity because they are involved in a creative challenge. Although this challenge does not guarantee learning, it certainly helps by furnishing an environment in which the students—not the author of the worksheet—choose the formulas and formulate the questions. Even if they are unable to answer these questions readily, at least they know what the questions are, which makes it easier to hear the answers as they surface in a group or class discussion or in a teacher's lecture. It is very difficult for students to hear the answers to questions that they do not have, let alone understand them.

The essence of Make These Designs is its reversal of a traditional activity. Instead of asking for the graph, given the equation, it asks for the equation, given the graph. This type of reversal is a very powerful tool in the design of effective problem-solving activities. In fact, it can be said that students do not have a full understanding of most mathematical topics if they cannot comfortably reverse them. They do not fully understand addition if they do not understand subtraction—otherwise how would they solve $x + 1.234 = 5.123$? They do not fully understand the distributive law if they cannot factor anything. For a stimulating essay on the importance of reversal in the learning of algebra, see Rachlin (1987). Understanding this role of reversal can lead to a rethinking of all of one's teaching.

SOME SAMPLES OF STUDENTS' WORK

The following are excerpts from students' writing on Make These Designs. They give a sense of students' thinking and understanding as they engage in solving the problems.

I didn't really have a method but I followed patterns once I found them. For instance in a design like a lot of parallel slanted lines, if I made one line, I'd keep the same value of x while changing the value of b.

In order to create the parallel, cross to the right, line pattern [**fig. 1c**], you must change the y-intercept, or b, because they all have the same slope, or m, which means that needs to be consistent or remain the same. By changing the y-intercept you change where the line is placed on the axis. So, each y-intercept increases or decreases by 2 or –2.

[**Fig. 1a**] $mx \rightarrow -mx$. I just started seeing what happened when I changed the m and not the b. The m changes, the b stays zero, and the m goes from 10–.9

In this picture [**fig. 1b**] the b in the formula $y = mx + b$ is negative four because all the lines cross the y-intercept at negative four. The m's range from five to negative five, five tenths of an increment away from the m before and after it. For every number possible for m, its reciprocal, negative and positive version are also possible values for m.

ASSESSMENT

I have often used the last figure as an extra-credit problem if the activity is done as an introduction to m and b. In contrast, I have used the last picture, or the whole sheet, late in the course to assess students' understanding of slope and x- and y-intercepts. If the activity is used as a wrap-up, students should write full explanations of how some of the figures were created. Explaining helps them cement their understanding, and it helps teachers assess it. To support students, give them a copy of the foregoing bulleted questions, which can serve as a content checklist for a written report.

EXTENSION

The activity can easily be extended. For example, students can create their own designs, which can be printed or shown on the overhead projector for others to emulate. In a second-year-algebra or precalculus class, students can be asked to do a similar activity using quadratic, polynomial, or trigonometric functions.

TECHNICAL NOTES

Originally, I used a graphing program that I had written in the Logo language (Picciotto 1990) for this activity. More recently, I have been using the TI-82 calculator. **Figure 1** was created with the latter. The given "window" makes each pixel be worth 0.2 both horizontally and vertically and draws axes with ticks that are one unit apart. These numbers may need to be adjusted for other electronic graphers, or a new set of designs can be created with whatever electronic grapher is available.

To get this many lines onto the screen may require using some of the special features of the calculator. On the TI-81, only four functions can be displayed at a time, so the line-drawing capability in the DRAW menu must be used. On the TI-82, ten functions can be graphed. To get more, students can use bracketed lists of parameters. See the respective technical manuals for more information.

CONCLUSION

Using technology does not accomplish miracles, but it does present an excellent context for the reversal of standard tasks, which yields powerful educational benefits. Still, the electronic grapher should not be the only way through which these concepts are addressed. Although the technology helps students' emerging understanding of the parameter-graph connection, this development should not be mistaken for a full understanding of linear functions and rate of change. For example, although the slope of a line in a Cartesian graph is a very important way to think about the rate of change of the corresponding function, it is only one possibility. Other representations—tables of values, so-called real-world situations, and such other visual representations as function diagrams and manipulatives—can also help students develop their understanding of these concepts. Do not put all your eggs in the technology basket!

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