Scaling Tables

1. With your neighbors, choose a simple shape drawn on graph paper, with area greater than 1. Scale it, and fill out the first two tables:

Scaling Factor	Perimeter
1	
2	
3	
4	
5	
x	

Scaling Factor	Area
1	
2	
3	
4	
5	
x	

Scaling Factor	Perimeter
1	
2	
3	
4	
5	
x	

Scaling Factor	Area
1	
2	
3	
4	
5	
x	

2. Repeat #1 with another shape of area greater than 1. Use the next two tables.

3. With your neighbors, choose a simple solid made of cubes, with volume greater than 1. Scale it, and fill out the first two tables:

Scaling Factor	Surface Area
1	
2	
3	
4	
5	
x	

Scaling Factor	Volume
1	
2	
3	
4	
5	
x	

Scaling Factor	Surface Area
1	
2	
3	
4	
5	
x	

Scaling Factor	Volume
1	
2	
3	
4	
5	
x	

4. Repeat #3 with another solid of volume greater than 1. Use the next two tables.

5. You should have eight formulas of the form $y = kx^n$. Enter them in a calculator or spreadsheet, and check that you have the correct values in your tables.

6. For each formula, what is k? what is n?

7. What value of n corresponds to perimeter? to area? to volume? Explain.

n^{th} Power Variation

The function $y = kx^n$ is called an n^{th} *power variation.*

1. For an n^{th} power variation, if $x = 0$, then $y =$ ____ . What does this tell you about the graphs of n^{th} power variations?

2. Choose your own n^{th} power variation equation, $y =$ __________, with both n and k different from your neighbors'. Fill out the table for your equation:

x	*y*
-3	
-2	
-1	
0	
1	
2	
3	
4	
6	
8	

3. Look for these patterns in your table. What happens to y when you multiply x by:
   a. 2? b. 3? c. 4?
   This is called the *multiply-multiply* pattern:
   For an n^{th} power variation, when x is multiplied by c, y is ____________________________

4. Find n and k for these n^{th} power variations. Sketch the graphs.

x	*y*
2	36
4	144
6	324
8	576

x	*y*
2	24
4	192
6	648
8	1536

x	*y*
2	10.2
4	20.4
6	30.6
8	40.8

x	*y*
2	31.6
4	126.4
6	284.4
8	505.6

Recognizing n^{th} Power Variation

Consider the n^{th} power variation $y = 5x^2$. If you multiply x by 3, (replace x by $3x$,) what happens to y?

$$y = 5(3x)^2 = 5 \cdot 9x^2 = 9 \cdot 5x^2$$

So the new y is 9 times the original y.

1. If you multiply x by c, what happens to y if $y = 5x^2$?

2. If you multiply x by c, what happens to y if $y = kx^n$?

This is called the *multiply-multiply* pattern. It only works *consistently* for n^{th} power variations.

3. Which of these is an n^{th} power variation? Try the *multiply-multiply* pattern. If it works *consistently* then it's an n^{th} power variation:

a.

x	***y***
2	-2
4	3
6	8
8	13

b.

x	***y***
2	12
4	48
6	108
8	192

c.

x	***y***
2	12
4	48
6	192
8	768

d.

x	***y***
2	2
4	16
6	54
8	128

4. Find the equations for each of the tables above. For the ones that are not n^{th} power variations, what are they?

STOP! Let's talk about roots and fractional exponents.

5. Find the equations for each of the tables below. They are n^{th} power variations (and thus have a *multiply-multiply* pattern), but n is not a whole number! (Some numbers are approximations.)

a.

x	***y***
1	1
4	2
9	3
16	4

b.

x	***y***
1	2
2	2.828
3	3.464
4	4

c.

x	***y***
1	1
3	1.442
9	2.080
27	3

d.

x	***y***
1	5
4	7.937
8	10
9	10.400

6. Sketch the graphs.

A New Meaning for Exponents

Surface Area of a Cube

1. If the surface area of a cube is 6, then its side length is 1. Complete the table by finding the side length of cubes with the given surface areas.

Surface Area	Side Length
6	1
24	
54	
60	

2. This is a *multiply-multiply* table. Explain.

3. Find the formula for the table. What is the value of k and what is the value of n?

A Fractional Exponent

4. Find x.
   a. $2^5 \cdot 2^5 = 2^x$
   b. $2^3 \cdot 2^3 = x^6$
   c. $(2^4)^2 = 2^x$

5. Find x.
   a. $9^x \cdot 9^3 = 9^6$
   b. $9^x \cdot 9^x = 9^2$
   c. $9^x \cdot 9^x = 9^1$
   d. $B^x \cdot B^x = B^1$

6. Find x.
   a. $(9^x)^2 = 9^6$
   b. $(9^x)^2 = 9^1$
   c. $(B^x)^2 = B^6$
   d. $(B^x)^2 = B^1$

7. The previous problems suggest a meaning for the exponent ½. Explain.

8. Using this meaning of the exponent ½, find (without a calculator, as much as possible):
   a. $16^{1/2}$
   b. $400^{1/2}$
   c. $25^{1/2}$
   d. $2^{1/2}$

9. Does it make sense to use the exponent ½ in the equation you found in Problems 3? Explain.

Laws of Exponents and Radical Rules

Rules for operations with radicals can be derived from laws of exponents using the fact that

$$x^{1/2} = \sqrt{x}$$

The following rules assume *a* and *b* are non-negative.

Exponent Rule	**Radical Rule**
$a^{1/2} \cdot a^{1/2} = a^1$	$\sqrt{a} \cdot \sqrt{a} = a$
$a^{1/2} \cdot b^{1/2} = (ab)^{1/2}$	$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$
$\dfrac{a^1}{a^{1/2}} = a^{1/2}$	$\dfrac{a}{\sqrt{a}} = \sqrt{a}$
$\dfrac{a^{1/2}}{b^{1/2}} = \left(\dfrac{a}{b}\right)^{1/2}$	$\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

10. Check all the radical rules by using a = 16 and b = 9.

Find the Formula

All the formulas on this sheet are linear, n^{th} power variations, square or cube roots. You can recognize them by looking for patterns in the tables.

1. Name the pattern, name the function, and write the general formula:
   a. When you add d to x, you add md to y
   b. When you multiply x by c, you multiply y by c^n
   c. When you multiply x by c, you multiply y by $\sqrt{c}$
   d. When you multiply x by c, you multiply y by $\sqrt[3]{c}$

2. If a car is going faster, it takes a longer distance to brake to a stop. The following measurements were collected for a certain car:

Speed (mph)	**Braking Distance (ft)**
10	5
20	20
30	45
40	80

   a. Find a formula for the braking distance as a function of speed.
   b. Assuming the same formula holds at greater speeds, figure out the braking distance for 65 mph.

3. A crate weighs more when you put watermelons in it. The following is the approximate weight of the crate depending on the number of watermelons.

# of Watermelons	**Weight (lbs)**
4	108
6	132
10	180
12	204

   a. Find a formula for the weight of the crate as a function of the number of watermelons.
   b. What is the weight of the empty crate?
   c. What is the average weight of a watermelon?

4. When the wind blows faster, windmills generate more power. The following approximate measurements were collected for a certain windmill:

Wind Speed (mph)	**Power (watts)**
3	3.24
6	25.92
9	87.48
12	207.36

   a. Find a formula for the power as a function of wind speed.
   b. Find the power generated by a 10-mph wind.

5. The longer a pendulum is, the longer its period. Here are some approximate measurements:

Length (cm)	**Period (s)**
10	.63
20	.90
40	1.27
80	1.79

   a. Find a formula for the period as a function of the length.
   b. How long is the period for a 30 cm pendulum?

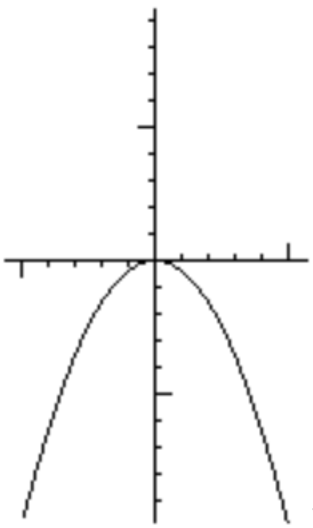
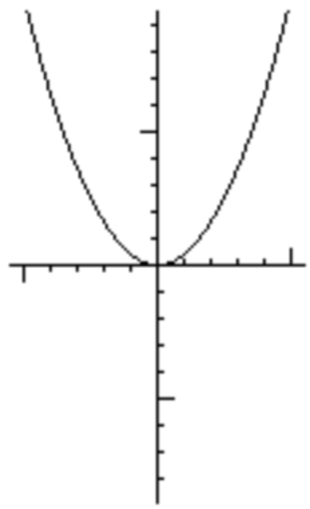
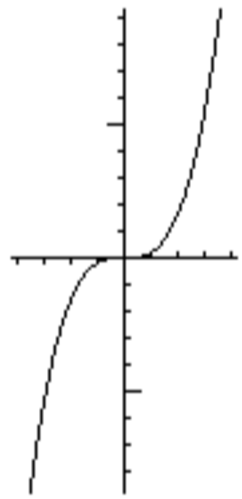
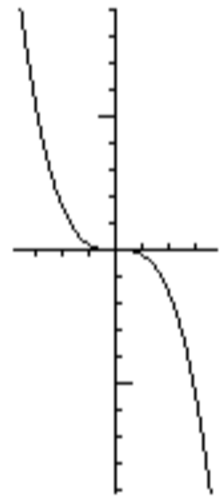
n^{th} Power Variation Graphs and Tables

$$y = kx^n$$

1. What can you say about the x- and y-intercepts of n^{th} power variation graphs?

2. For an n^{th} power function, when $y = k$, $x =$______. Explain why this works algebraically.

3. Find values of n and k that yield graphs with the following four basic shapes:

a.

b.

c.

d.


4. Explain the *multiply-multiply* property of n^{th} power variation equations
   a. Using $y = 4x^2$ as your example.
   b. With an algebraic explanation.

5. Find n and k for these n^{th} power variations.

a.

x	***y***
-4	-480
-2	-60
-1	-7.5
0	0
2	60
4	480
8	3840

b.

x	***y***
-8	2730.7
-4	170.67
-2	10.667
0	0
4	170.67
8	2730.7
16	43690

c.

x	***y***
-4	-128
-2	-32
-1	-8
0	0
2	-32
4	-128
8	-512

Inverse n^{th} Power Variation

An *inverse n^{th} power variation* has an equation of this type: $y = k / x^n$

1. Among the following tables, look for the following patterns: add-add (linear function), multiply-multiply (n^{th} power variation), and multiply-divide (inverse n^{th} power variation).

a.

x	y
2	0.8
4	3.2
6	7.2
8	12.8

b.

x	y
2	1.2
4	7.4
6	13.6
8	19.8

c.

x	y
2	7.071
4	10
6	12.247
8	14.142

d.

x	y
2	1.5
4	.75
6	.5
8	.375

2. Find a formula for each function.

3. Make tables for these functions:

a. $y = 60 / x^2$

x	y

b. $y = 8 / x^3$

x	y

4. Describe the *multiply-divide* pattern in tables 1d, 3a, 3b.

5. Graph these three functions and sketch the graphs.

6. These three tables have a *multiply-multiply* pattern, but n is not a positive number!
   a. Find n for each one.
   b. How is n related to the equation of the function?

7. The further you are from the center of the earth, the less you weigh. The following is the weight of a certain astronaut at various distances from the center of the earth. The earth's radius is approximately 4000 miles.

Distance (miles)	**Weight (lbs)**
5000	96
7500	42.67
10,000	24
15,000	10.67

a. Find a formula for the astronaut's weight as a function of the distance from the center of the earth.
b. How much does this astronaut weigh on earth?