Chapter 4 Simplifying Algebraic Expressions

The main purpose of this chapter is to lead the students in a discovery of the correct handling of addition and subtraction of expressions that include variables.

New Words and Concepts

This chapter focuses on adding and subtracting **monomials** and **polynomials**, in particular the distinction between parentheses that are preceded by a plus sign and parentheses that are preceded by a minus sign. (The full treatment of parentheses that are preceded by a multiplication symbol is left for Chapter 5.) With the Lab Gear, combining "unlike" terms is a very unusual mistake. The students practically teach themselves that only "like" terms can be combined.

Teaching Tips

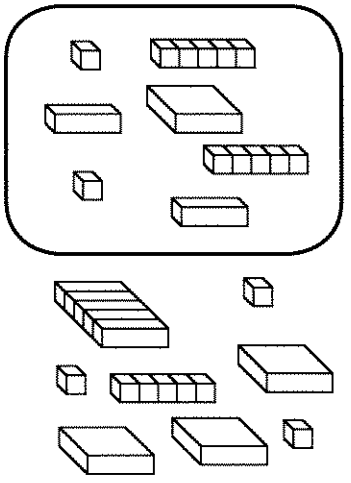
Encourage students who discover good techniques for finding the perimeters and surface areas of given figures to share them with their classmates. The best strategy is to use subtraction in order to find the length of a side, or the area of a face where two blocks make contact.

Lesson Notes

- **Lesson 1,** Polynomials, page 45: You may need to have some students use the overhead projector to remind the class how to simplify upstairs blocks in the minus area. See Chapter 3, Lesson 5.
- **Lesson 2,** Removing Parentheses, page 47: Insist on seeing each equation represented with blocks. It is, of course, faster to work out the arithmetic than to set up the blocks, but working out the visual layout of each expression will guarantee a deeper understanding.
- **Lesson 3,** Adding Polynomials, page 48: This should be quite straightforward, again a review of combining like terms. If your students really understand this, only assign a few problems from this lesson.
- **Lesson 4,** Subtracting Polynomials, page 49: Make sure the students understand both methods ("taking away" and "adding the opposite").
- **Lesson 5,** More on Parentheses, page 51: Removing parentheses incorrectly is a source of many errors. The rule that this lesson leads to should be memorized. But just knowing the rule is not enough. Insist that the students be able to explain their answers to the problems in this lesson in terms of the blocks.

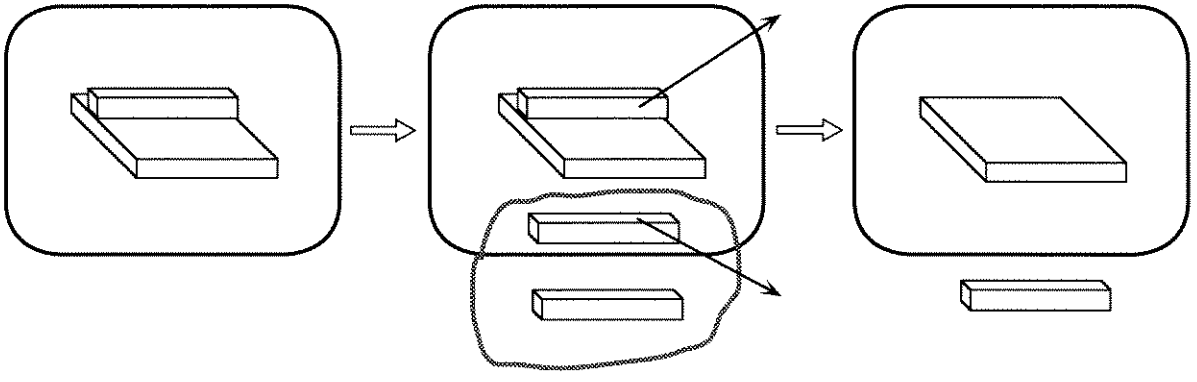
Polynomials

Algebraic expressions like $2x$, or $-3x^3$, or $2xy$ are called **monomials**. Sums of monomials are called **polynomials**.



1. Write the quantity that this figure shows.
2. What quantity does it show if $x = 2$? Use substitution to find out. Replace each x-block with 2. What should you replace the x^2-block with? What should you replace the $5x$-block with? Cancel what you can and write the answer.
3. While working the above problem, you may have thought about *simplifying* the expression *before* doing the substituting. For example, the x^2 in the minus area can be cancelled along with one of the x^2 outside. What else can be cancelled? Shade the blocks that can be removed, and write the simplified polynomial.
4. Use the simplified expression from problem 3. What does it show if $x = 2$? Use substitution to find out, and write your answer. This should be easier to do, and you should get the same answer. (If not, find your mistake!)

As you know, to simplify upstairs blocks in the minus area, you can use the adding zero trick.

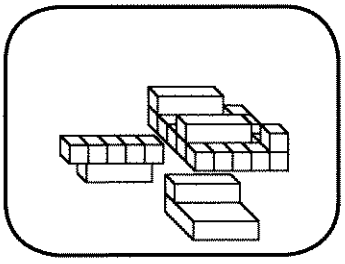


5. Write the quantity shown in this figure before and after it is simplified.

For each figure:

a. Copy the figure with your blocks.
b. Simplify the figure using the blocks.
c. Write the polynomial that names the original blocks.
d. Write the simplified polynomial.

6.



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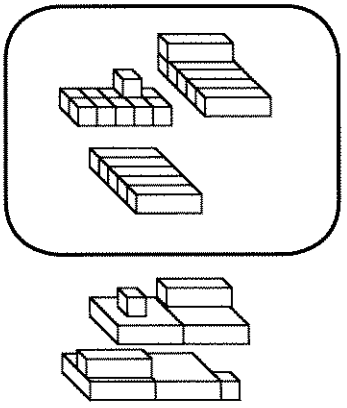

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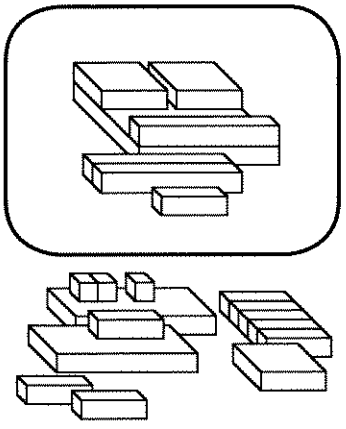

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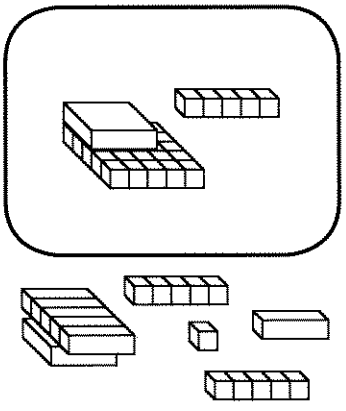

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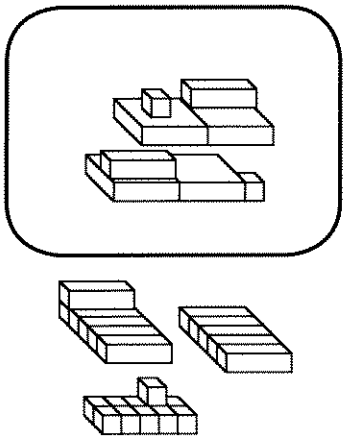


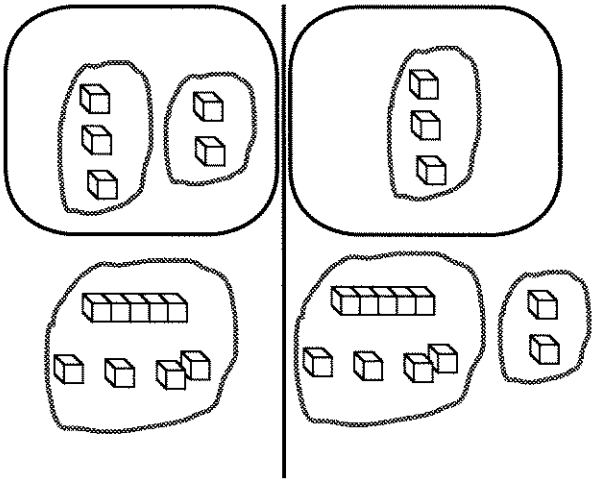
Removing Parentheses

When an algebraic expression is enclosed in parentheses, you should think of it as one unit.

1. For example, if you write 9 – (3 + 2), you are saying *subtract the quantity 3 + 2 from the number 9*. Write the result.

Be careful! If you remove parentheses from an expression, you may change its value without intending to.

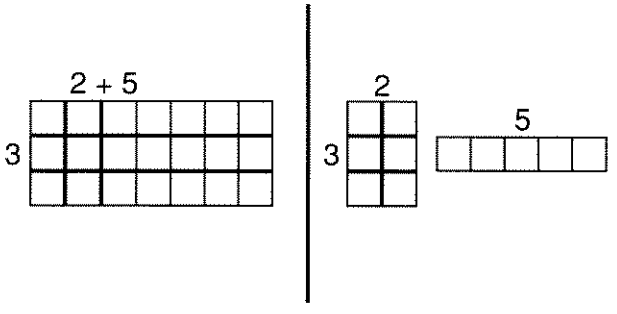
2. For example, this figure compares 9 – (3 + 2) and 9 – 3 + 2. Are the two sides equal? Use blocks and simplify to find out.



$9 - (3 + 2) \stackrel{?}{=} 9 - 3 + 2$

For these problems, use the Lab Gear to help you decide if the two sides are equal. Write T if they are equal, F if they are not equal. The figure for problem 3 is shown to get you started.

3. $3\,(2 + 5) = 3 \cdot 2 + 5$
4. $4 + (6 - 2) = 4 + 6 - 2$
5. $4 - (1 + 6) = 4 - 1 + 6$
6. $8 - (3 - 5) = 8 - 3 - 5$
7. $6 - (4 - 3) = 6 - 4 - 3$
8. $2\,(4 - 7) = 2 \cdot 4 - 7$
9. $8 - (2 + 5) = 8 - 2 + 5$
10. $10 + (3 + 8) = 10 + 3 + 8$
11. $5\,(-5 + 2) = 5\,(-5) + 2$
12. $-2\,(4 - 3) = -2 \cdot 4 - 3$
13. $5\,(4 - 2) = 5 \cdot 4 - 2$
14. $3 + (2 - 4) = 3 + 2 - 4$



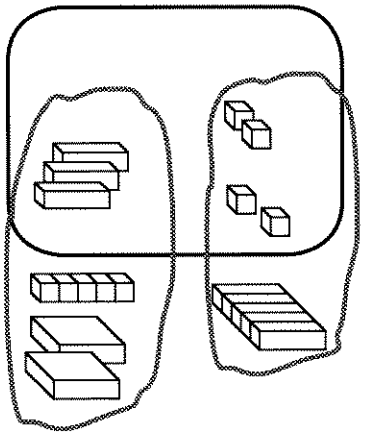
15. Copy and complete this sentence:

    Parentheses that are preceded by _____ can be removed without changing the value of an expression.

    Check whether you are right by creating more problems of this type. Discuss them with a classmate.

Adding Polynomials

This figure shows $(2x^2 - 3x + 5) + (5x - 4)$



1. Write the sum. Don't forget to cancel what you can.

Use the Lab Gear to add these polynomials.

2. $(x^2 - 4x + 2) + (6x + 5)$
3. $(2y^2 + 3y + 25) + (2y - 10)$
4. $(-y^2 + 2x^2 - 3y) + (y + 5x - x^2)$
5. $(4x^2 - 2xy + 10) + (y^2 - 2x^2 + 3xy + 5y)$
6. $(2x^2 - 2x - 1 + 3xy) + (2y^2 - 3x + 6 + x^2 - xy)$
7. $(x^2 + 7x + 6) + (3x^2 + 2x - 5) + (-x^2 - 3x + 4)$

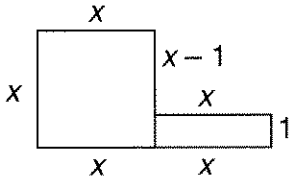
☑ Self-check

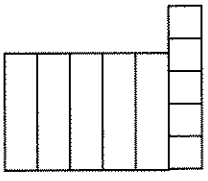
2. $x^2 + 2x + 7$
6. $3x^2 + 2y^2 + 2xy - 5x + 5$

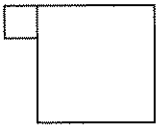
Exploration 1 Perimeter

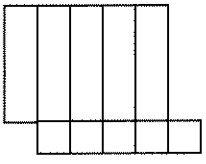
In problems about perimeter, area, and volume, assume that x and y are positive. In fact, assume that $1 < x < 5 < y < 10$.

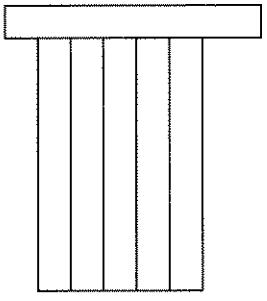
Find the perimeter of these figures.

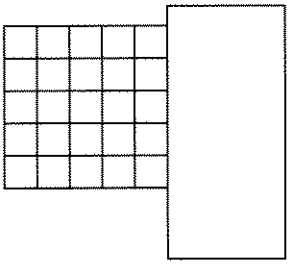
1. 

2. 

3. 

4. 

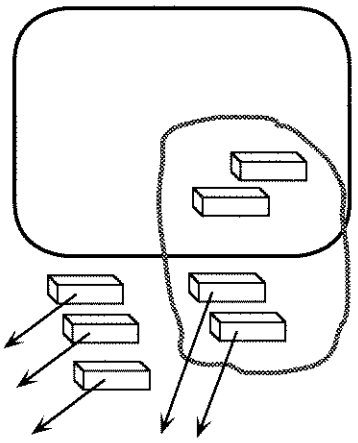
5. 

6. 

Subtracting Polynomials

To calculate $3x - 5x$, you can *put on* $3x$, *add zero*, and *take off* $5x$.

1. Write the answer to this problem.

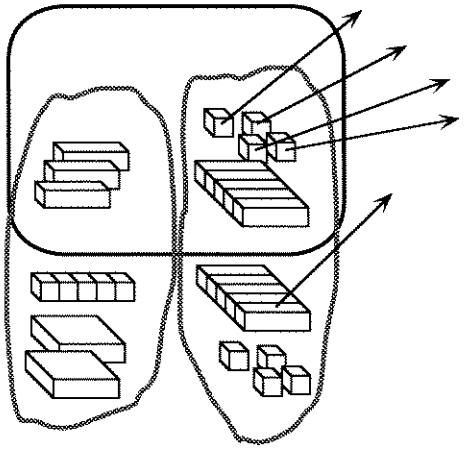


Practice with these problems. Not all of them can be simplified.

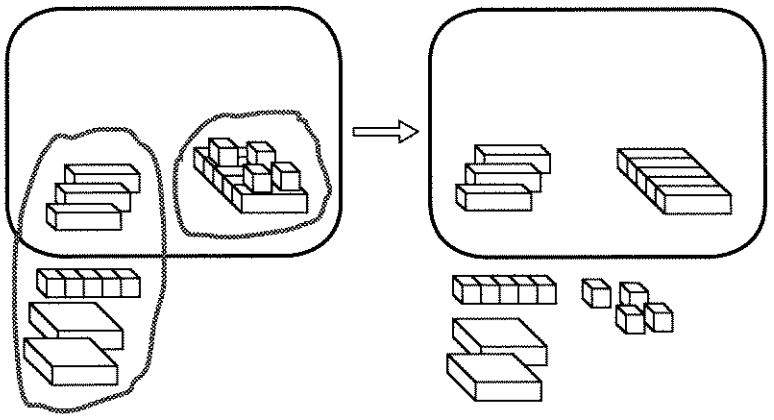
2. $2y - 7y$
3. $3xy - (-2xy)$
4. $x^2 - 4x^2$
5. $2xy - 2x$
6. $-5x - 4x$

To calculate $(2x^2 - 3x + 5) - (5x - 4)$, you can *put on* the first polynomial, and *take off* the second, after *adding zero* as necessary.

7. Explain the figure. Write the answer.



Look at $(2x^2 - 3x + 5) - (5x - 4)$ again. Another approach to subtracting is to put the whole quantity $5x - 4$ inside the minus area and simplify.



8. Explain what was done to simplify this expression.

9. Copy and complete this sentence: With both methods, to subtract $(5x - 4)$, we ended up adding _____.

Use the Lab Gear to solve these subtraction problems.

10. $(x^2 - 6x + 4) - (2x - 3)$

11. $(2y^2 + 5y - 2) - (y + 8)$

12. $(y^2 - 3y - 7) - (y^2 + 2y - 3)$

13. $(-4x^2 + 8x - 25) - (3x^2 - 20)$

14. $(2xy + y^2 + 6) - (4y^2 - xy + 5 - x)$

15. $(3x^2 - 2x - 1 + xy) - (2y^2 - 3x + 6 + x^2 - xy)$

16. $(4x^2 - 2xy + 10) - (y^2 - 2x^2 + 3xy + 5y)$

17. $(-y^2 + x^2 - 3y) - (-2x^2 + y - 2y^2 + 5)$

18. $(x^2 + 5x + 6) - (3x^2 + 2x - 7) - (-2x^2 - 3x + 4)$

☑ Self-check

10. $x^2 - 8x + 7$
16. $6x^2 - y^2 - 5xy - 5y + 10$

More on Parentheses

Use the Lab Gear to help you decide which of the expressions a, b, c, or d are equal to the expression on the left. Explain your answers.

1. $x-(5+2x)$
   a. $x-5+2x$
   b. $x-5-2x$
   c. $x+5+2x$
   d. $x+5-2x$

2. $y-(6+3y)$
   a. $y-6+3y$
   b. $y-6-3y$
   c. $y+6+3y$
   d. $y+6-3y$

3. $2x-(-4+3x)$
   a. $2x-4+3x$
   b. $2x-4-3x$
   c. $2x+4+3x$
   d. $2x+4-3x$

4. $3y+(5-2y)$
   a. $3y-5+2y$
   b. $3y-5-2y$
   c. $3y+5+2y$
   d. $3y+5-2y$

5. $x-(7-2y)$
   a. $x-7+2y$
   b. $x-7-2y$
   c. $x+7+2y$
   d. $x+7-2y$

6. $6x-(-3-x)$
   a. $6x-3+x$
   b. $6x-3-x$
   c. $6x+3+x$
   d. $6x+3-x$

Find a way to write an equivalent expression without parentheses.

7. $2x^2-(4-x-x^2)$

8. $(2x^2-4)-(x-x^2)$

9. Explain how you can correctly remove parentheses from an algebraic expression when there is a minus sign before the parentheses.

Exploration 2 Which is Greater?

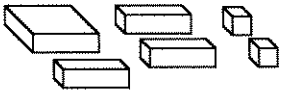
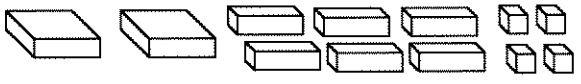
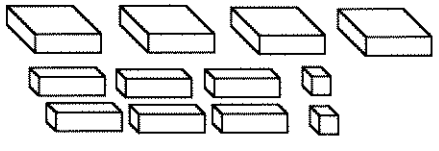
For each of these problems, represent both expressions with the Lab Gear. Simplify the two expressions, and then decide which side is greater, whether they are equal, or whether it is impossible to tell. Write =, >, <, or ?.

Which is greater…

1. $5x + x^2 + 1 - 2 - (2x^2 + 2x - (x^2 + x + 1))$ or $3x + 3 - 5 - (2x - (x + 2))$?
2. $4 + 10x - 2x - (3x + 2 - 1)$ or $5x + x^2 - 1 - (x + x^2 + 4 - x)$?
3. $7x - x^2 - (2x - (x + x^2))$ or $6x + 5 + x^2 - (x^2 + 5 - (2x^2 + 3))$?
4. $6x + 5 - (2x + 2) - (2x + 5 - 2)$ or $2x + 1 + x^2 - 4 - (x^2 - 3)$?
5. $5x + 8 - 2 - (3x + 5 - 2)$ or $3x + 7 + x^2 - 1 - (4 + x^2)$?
6. $10x + 2 - 5 - (3x + 1 - 4)$ or $7x + x^2 - x - (3x^2 - (2x^2 + x))$?

Exploration 3 Make a Rectangle

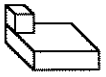
For each problem, arrange the given blocks into a rectangle or square. Sketch it (as seen from above) and write a multiplication equation relating the length, width, and area of the figure. Some have two solutions (find them both).

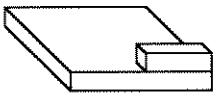
1. 
2. 
3. 

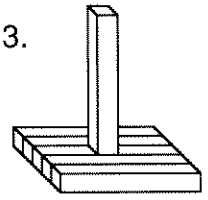
4. $x^2 + xy + x + y$
5. $x^2 + 7x + 10$
6. $3x^2 + 5x + 2$
7. $6x^2 + 7x + 2$
8. $6x^2 + 19x + 10$
9. $3x^2 + 16x + 5$
10. $4x^2 + 8x + 4$

Exploration 4 Surface Area

Imagine these buildings are made by gluing Lab Gear blocks together. The surface area is the total area of all the exposed faces, even the bottom of the building. Find the surface areas.

1. 

2. 

3. 

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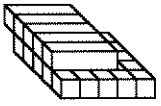
5. 

Exploration 5 Always, Sometimes, or Never?

Write A, S, or N for each of these statements, depending on whether it is always, sometimes, or never true. Use the Lab Gear to help you decide. Explain your answers.

1. $x(3 + 2) = x \cdot 3 + 2$
2. $x^2 - (2 + 7) = x^2 - 2 + 7$
3. $y(y - 3) = y^2 - 3$
4. $5 + (3 - x) = 5 + 3 - x$
5. $6 - (y - 4) = 6 - y - 4$
6. $4 + (x + 2) = 4 + x + 2$