Inequalities in Two Variables Teacher Notes

This unit extends the graphical understanding of systems of equations to inequalities.

1A. Postcards and Letters

This activity creates a foundation for the graphical analysis of inequalities in two variables by first developing an understanding based on numbers and money. Make sure the students understand the introductory paragraph, especially the constraints: Shannon must send 16 to 18 postcards and letters, and she cannot spend more than $7.50 on stamps.

Allow calculator use.

As they work on #2, make sure that students' entries in their tables do satisfy the constraints. When almost everyone has found six to ten possible entries, ask them to double-check their work, and then to check their neighbors' as suggested in #3. The most likely errors are in the "total cost" column.

The heart of this lesson is in #4, where students are expected to plot not only their own points, but all the combinations that satisfy the constraints. They can start with the ones in their own table, and the ones in their neighbors', but they should not stop until they have found all 33 combinations. You might hint the (x,y) pairs they are looking for cannot (for example) be near the origin, as that would mean too few letters and postcards. Nor can they be near the top right of the graph, as that would mean too many. Correct answers will cover one compact area of the graph.

1B. Postcards and Letters

In this lesson, students make the transition from numbers and graphs to the symbolic representation, and the fact that lines form the boundaries of the feasible region.

Note that the inequality for "a non-negative number of letters" is $y \geq 0$, and the graph of the boundary line is the x-axis.

2. Graphing Inequalities in Two Variables
3. Systems of Inequalities in Two Variables

These lessons generalize the concept of using graphs to represent inequalities in two variables. In the first lesson, there is a single inequality; in the second we have systems of two inequalities.

Note that there is no straightforward way to know which side of the line should be shaded. However it is sufficient to test the coordinates of one point to see if it is on the right side.

1A. Postcards and Letters

Shannon, a math teacher, is travelling. She didn't bring her computer or her phone. She wants to send postcards or letters to her friends. In addition to 16 friends to whom she absolutely must write, there are 2 friends to whom she may write. She already has all her supplies, plus $7.50 to buy stamps. She will write only one postcard or letter for each friend.

Assume that postcard stamps cost 35 cents, and letter stamps cost 50 cents.

1. a. Does she have enough money to buy stamps for 18 letters?

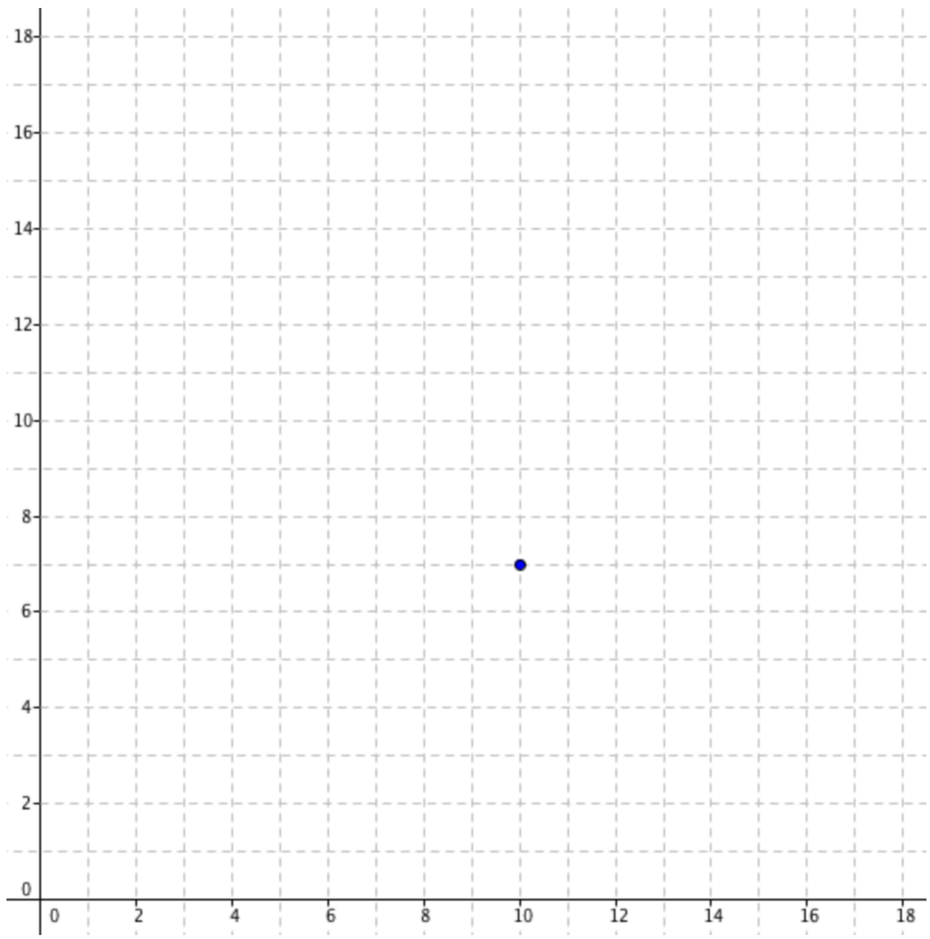
   b. Does she have enough money to buy stamps for 18 postcards?

2. Find combinations of postcards and letters that satisfy the constraints outlined above. One has been done for you and added to this table. It satisfies the constraints, as she is sending a total of 17 postcards and letters, and she spends $7.00 on stamps.

Postcards		Letters		Postcards and Letters	
Number of Postcards	**Cost of Postcard Stamps**	**Number of Letters**	**Cost of Letter Stamps**	**Total Number**	**Total Cost of Stamps**
10	10 · 0.35 = $3.50	7	7 · 0.50 = $3.50	10+7=17	3.50+3.50=$7.00

3. Check whether a neighbor's combinations do satisfy the constraints: is Shannon sending an acceptable total number of letters or postcards? Can she afford the stamps?

4. Say that x = the number of postcards, and y = the number of letters. Graph all possible combinations of x and y given the above constraints by marking each possible (x,y) combination with a dot. The dot for 10 postcards and 7 letters has already been marked for you. If you do it correctly, there should be 33 dots, including some on the x-axis. **Hint**: the dots will all be next to each other, in one part of the graph.



1B. Postcards and Letters (continued)

1. Create inequalities (the first one is done for you) involving x and/or y that express that Shannon will:

Words	Inequalities
send 16 messages or more	$x + y \geq 16$
send 18 messages or fewer	
spend $7.50 or less on stamps	
send a non-negative number of letters	
send a non-negative number of postcards	

2. Shannon may send exactly 16 messages (the total of letters and postcards).
   a. Write an equation (not an inequality) that corresponds to these words.

   b. Graph that equation on the same graph as the dots you plotted in the previous lesson.
   c. How is that graph related to the dots?

3. Each of the inequalities that you created in question 1 has an associated equation. Complete this table to show the equation associated with each inequality. The first one has been done for you.

In words	Inequality	Associated Equation
send 16 messages or more	$x + y \geq 16$	$x + y = 16$
send 18 messages or fewer		
spend $7.50 or less on stamps		
send a non-negative number of letters		
send a non-negative number of postcards		

4. Graph the five equations that you found in the previous problem. Label each line with its equation.

5. Describe where on your graph you would find an (x,y) pair that corresponds to:
   a. too few messages sent

   b. too many messages sent

   c. too much spent on stamps

   d. exactly $7.50 spent on stamps

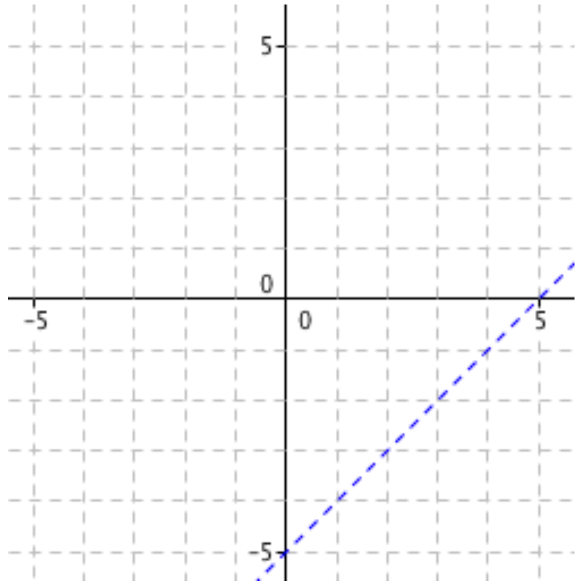
6. What is the largest possible number of letters she could send?

7. How many postcards and letters should she send if she wants to spend:
   a. As little as possible on stamps?

   b. **Challenge:** exactly $7.50 on stamps?

2. Graphing Inequalities in Two Variables

You know how to graph an equation such as $x - y = 5$. Here is the graph:



1. Choose values of x and y so that the inequality $x - y < 5$ is true. (Choose values different from your neighbors.)

2. Put dots in the figure for the coordinates (x, y) that you and your neighbors found in #1. If you did this correctly, all the dots should be on the same side of the line.

3. a. Choose a point on the other side of the line, different from your neighbors.
   b. For that point, is the inequality $x - y < 5$ true?

   c. What about for the points chosen by your neighbors?

4. Lightly shade the part of the plane on the side of the line where $x - y < 5$.

5. Do points on the line satisfy the inequality?

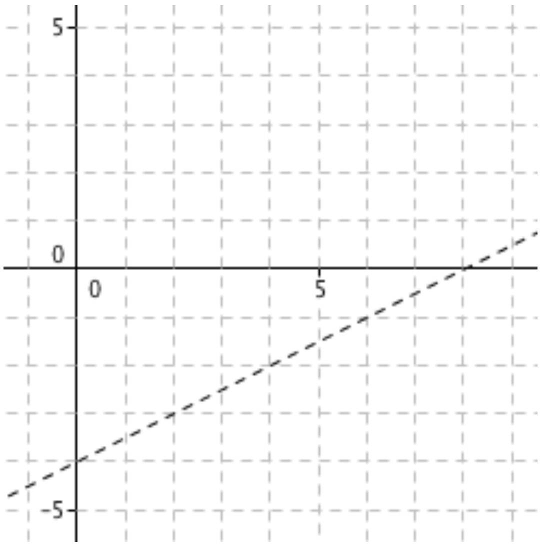
The area you shaded is called a *half-plane*. It is the graph of the inequality. The line is not included in that graph. The fact that the line is dotted rather than continuous represents the fact that points on the line do not satisfy the inequality.

6. Graph the inequality $5x + 2y \geq 10$. In this case the line at the edge of the half-plane should not be a dotted line, because points on the line do satisfy the inequality.

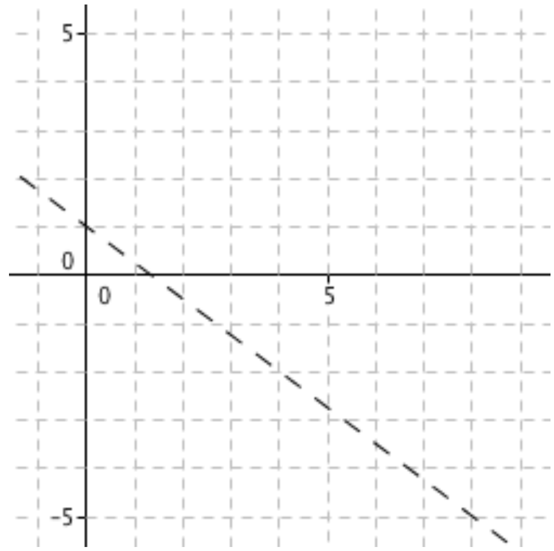
7. Graph the inequalities:
   a. $x - 2y > -4$
   b. $3x + 4y \leq -12$

3. Systems of Inequalities in Two Variables

Here is the graph of $x - 2y = 8$.

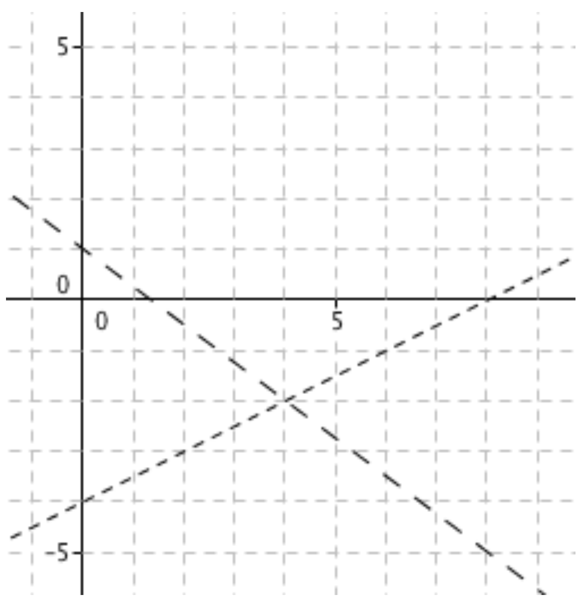


And here is the graph of $3x + 4y = 4$



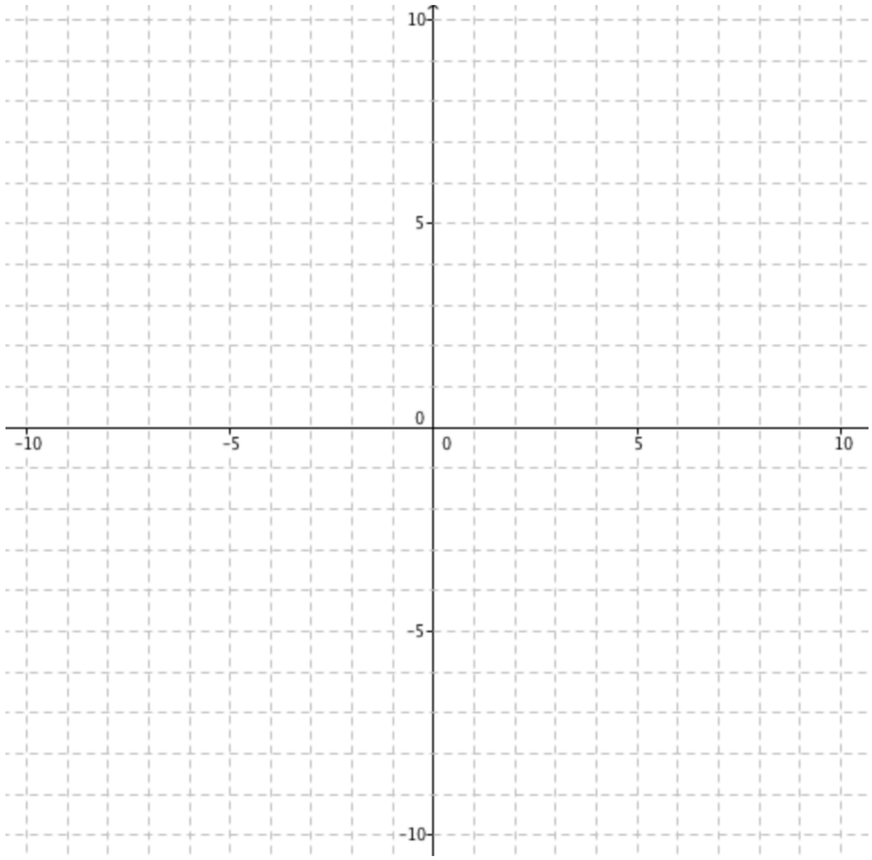
1. Using the figures above, graph
   a. $x - 2y > 8$
   b. $3x + 4y \leq 4$
   If the boundary line is included, remember to make it a continuous line.

2. In the figure below, shade in the area where *both* inequalities from #1 are true.



You have graphed this *system of inequalities*: $\begin{cases} x - 2y > 8 \\ 3x + 4y \leq 4 \end{cases}$

3. Graph the system: $\begin{cases} x - 2y > -4 \\ 3x + 4y \le -12 \end{cases}$



Solving a system of linear equations in two variables means finding one or more (*x*,*y*) pairs that satisfy both equations. Solving a system of inequalities in two variables means finding the (*x*,*y*) pairs that satisfy both inequalities. You show the solutions in a graph.

4. Solve this system of inequalities in two variables: $-4x + 5y < 20$ and $2x + y > -6$