Systems Teacher Notes

This unit is about systems of linear equations. The purpose is threefold:

◊ Reviewing basic ideas about equations and graphs

◊ Developing an understanding about the mathematics underlying the topic

◊ Learning solving techniques

Separating the third purpose from the first two is not likely to work well in terms of retention. Any direct instruction about solving techniques is more likely to be effective if it is built on a foundation of understanding. That foundation includes work with numerical examples in familiar contexts, as well as connections between tables, graphing, and symbol manipulation.

1A. At the Laundromat

This is a "real world" two-variable problem based on a situation involving money. In addition to providing context, it offers a number-based introduction to the topic, and a step-by-step introduction to creating equations for these types of word problems.

Before tackling the worksheet, ask students for different ways to make change for a dollar, and write those ways on the board. Then ask them for ways to make change for a dollar with only nickels and quarters, and keep track of their suggestions on the board in a table. Once it's clear most of them know what's going on, tell them the worksheet is about making change for five dollars.

When wrapping up the lesson, the big point you want to make is that there are two variables, and that they are related to each other. If Dan gets more nickels, he gets fewer quarters, and vice-versa.

If not all students understand #5, they get another chance at the beginning of the next lesson.

1B. At the Laundromat (continued)

This lesson involves going back to the table made in lesson 1A. Do not teach students how to solve equations to answer the questions: that is the purpose of this whole unit, and it cannot be learned instantly. Instead, encourage trial and error and calculator use to answer #3. That sort of work is useful to strengthen number sense, and to clarify what goes on when working with two variables.

The purpose of the lesson is to absorb the idea that on the one hand, there are many (x,y) pairs that satisfy $5x + 25y = 500$, but that on the other hand, adding another constraint can narrow down the search.

It is not necessary that every student solve #5, especially if you're short on time.

#6 is an application of the same sort of thinking as students did in the Laundromat lesson, without the support of a pre-made table. It can be done at any future time.

2. The Vanpool

This is another "real world" context, again involving money, but this time we will emphasize graphing.

Important: the variables stand for the fares, not for the number of people.

Students are not expected to make the graphs by first doing algebra. The next lesson is all about graphing these sorts of equations. All they need to do in this lesson is use common sense and simple calculations to find suitable (x,y) pairs, and plot those. That should yield three lines in #1-3 and one in #5. In #4, do not ask for $y = mx + b$ equations. Students should write equations related to the problem, such as $x + y = 12$ for #1.

3. Graphing *Ax* + *By* = *C*

Students will make their graphs on a separate piece of paper.

The main idea here is that when a linear equation is written in this format, it is not difficult to find the x- and y-intercepts, and therefore to graph the line. However, do not start by telling this to the students. Instead, start by making sure everyone knows the meaning of the intercepts, by asking questions like "If a point is on the x-axis, what is its y coordinate?" Then have students work on #1-5. Keep it interactive, offering nudges to students who need them, encouraging students who are able to do it to help their neighbors, and so on.

At the end of that process is a good time to summarize what they learned by having students contribute their strategies to a whole-class discussion, and of course by contributing your own explanations. At that time, you may also have a whole-class discussion to prepare students for #7, but use different numbers.

Finally, you can have students work on #6-8 on paper, which will be their opportunity to summarize the key ideas of the lesson.

Problems #9-11 offer an extension to the lesson. If you don't have time for them as part of the same period, you can save them to do for review at some future time. #9 is an opportunity to practice the big idea (finding the intercepts as a strategy for graphing.) #10 previews some ideas that will return in future lessons.

#11 is quite challenging. You should not expect everyone to be able to do it, but it may be a worth-while exploration for your strongest students.

4. Two Unknowns

This lesson builds on students' familiarity with linear equations in one variable. Each problem starts with an equation with two unknowns, with an extra constraint. The constraint provides a way to get down to a single unknown, and therefore to solve the equation.

Students may forget that in each case, we are looking for an (x,y) pair, not just an x, so you may need to keep reminding them of this as they work. It is also a good idea for them to check their final answer in the original equation, and to confirm it satisfies the constraint.

Note that #6 and #10 are slightly trickier, as the students will need to do a simple algebraic manipulation before making the needed substitution.

5. Mind Reading

There is probably no need for an introduction to the lesson. Have students start on the worksheet right away. Of course, be available to help students who need help, encourage students to help each other, and if needed, stop the work for a whole-class discussion.

Students may solve #1-2 by trial and error, by the method introduced in the previous lesson, or some other way. Have them share their approaches. #3-7b review the idea of equivalent equations, which is very basic to all of algebra, and particularly useful to support further work with systems.

#7c is challenging because the problem does not specify which two containers are removed, so it's really three problems in one.

6. Line Intersections
7. Lines and Systems
8. How Many Solutions?

These lessons are about the graphical representation of systems. Instead of rushing to the idea that solving the system is looking for the intersection of two lines, we take some time to break this down into bite-sized ideas: what does it mean for a point to satisfy two equations? What about the case of equivalent equations? What if the lines are parallel?

9A. Combining Equations

This lesson is intended to work with an electronic grapher that readily graphs equations in the form $Ax + By = C$. (For example: GeoGebra.) If students do not have access to such technology, the lesson can be done as a whole-class activity, with the teacher projecting the graphs, students sketching the graphs on paper, and answering the questions on the worksheet.

Introduce the lesson by asking for equations of horizontal and vertical lines, to make sure all students are reminded of what those look like.

Note that the goal of the activity is not so much the solving of the systems, as the system is solved as soon as the two lines are graphed. The goal is to see that combining the equations yields lines through the intersection of the graphs. In other words, it does not change the solution of the system.

This lesson is also described in this blog post:
http://blog.mathedpage.org/2011/04/add-till-its-plaid.html

9B. Combining Equations

This lesson applies the technique explored in the previous lesson: combining equations is a way to get down to a single variable. This time the work is done without the graph.

10. Using Systems

These are word problems that can be solved with the help of systems of equations.

11. Systematic Practice

You can use exercises from this lesson as homework or as items on assessments.

1A. At the Laundromat

Dan needs nickels and quarters to do his laundry at Science and Math Quick Wash. He has a five-dollar bill. The table shows one possible combination of coins he might get if he asks for nickels and quarters only.

Nickels		**Quarters**		**Total Coins**	
Number	**Value (cents)**	**Number**	**Value (cents)**	**Number**	**Value (cents)**
45	225	11	275	56	500
					500
					500
					500
					500
					500
					500
					500
					500
					500
					500
					500
x	$5x$	y			500

1. Add at least six more possibilities to the table and comment on any patterns you notice. (If you don't notice any patterns, add more possibilities until you do.)

2. a. What is the smallest number of coins Dan might get?

   b. What is the greatest number of coins Dan might get?

3. a. Would it be possible for the total number of coins to be an even number?

   b. Would it be possible for the total number of coins to be an odd number? Explain.

4. Would it be possible for Dan to have the same number of quarters as nickels? If so, how many of each would he have?

5. In the last row of the table, x represents the number of nickels.
   a. What does y represent?

   b. Explain the meaning of the expression $5x$ in the table.

   c. Fill out the rest of the row in terms of x and y.

1B. At the Laundromat (continued)

If Dan gets x nickels and y quarters, the entry in the table would look like this:

Nickels		Quarters		Total Coins	
Number	**Value (cents)**	**Number**	**Value (cents)**	**Number**	**Value (cents)**
x	$5x$	y			500

1. Complete the row, giving the total number of coins and their value in terms of x and y.

Any possible whole number pair of values (x,y) for the number of nickels and quarters that Dan might get in change will satisfy this equation:

$$5x + 25y = 500$$

For example, it is easy to show by substitution that the pair (45, 11) satisfies this equation:

$$5(45) + 25(11) = 500$$

This pair also satisfies this equation:

$$x + y = 56$$

The equation provides an extra *constraint* on x and y. A constraint is a condition that restricts the number of possibilities.

2. Is there another (x,y) pair that satisfies the same equation and the same constraint? If so, what is it?

3. Find an (x,y) pair that satisfies both the equation $5x + 25y = 500$ and the constraint given. (You may want to extend the table you made in Lesson 1A. You can save work by looking for patterns in your table.) Some may not be possible.
   Constraints:
   a. the total number of coins is 80.
   b. there are 20 times as many nickels as quarters
   c. there are 12 more nickels than quarters
   d. there are 8 more quarters than nickels

4. Each of the constraints in problem 3 can be expressed as an equation in x and y. Write each equation.

5. At Science and Math Quick Wash, the machines take 3 quarters and 1 nickel to wash and 1 quarter to dry. If Dan wants to do as many loads as possible:
   a. How many loads of wash will he be able to do?

   b. What change should he request for his five-dollar bill?

6. **Exploration**: Some dimes and quarters have a total value of $3.95. How many of each coin might there be?
   a. Find all the possibilities.

   b. What is the smallest possible number of coins?

   c. What is the largest possible number of coins?

   d. Explain your method of thinking about this problem and comment on any patterns you notice.

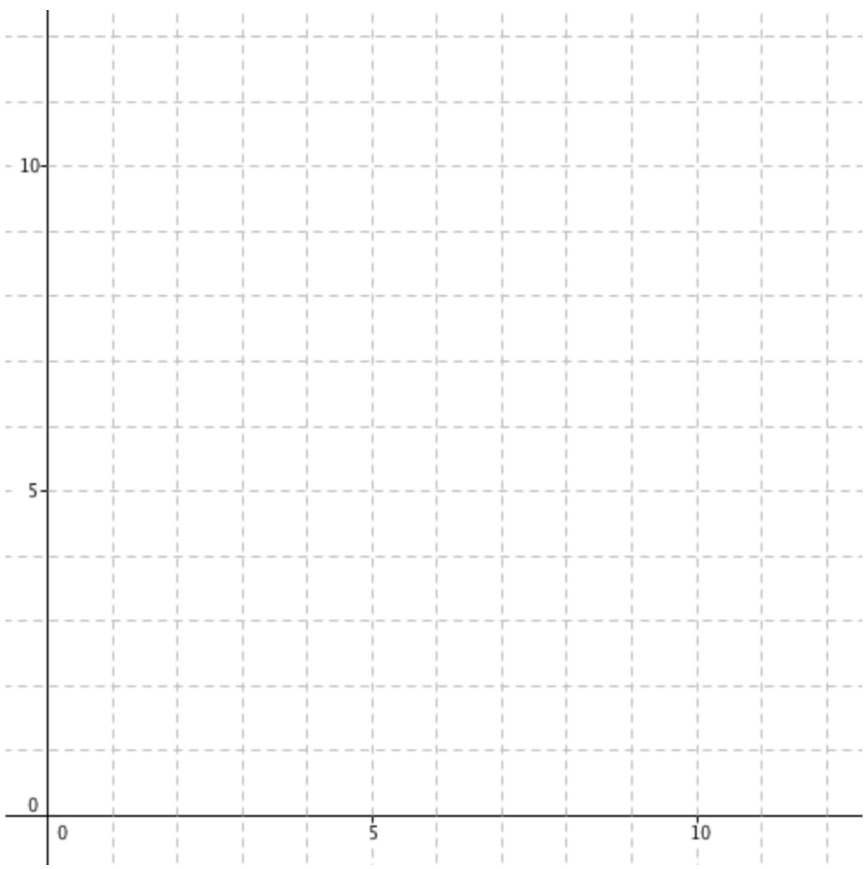
2. The Vanpool

A group of people decided to organize a vanpool to get to and from work and school. They estimated mileage costs to be about \$12.00 per day round trip. When they discussed how to share costs, they agreed that children and adults might have different fares.

In the following problems, let x stand for a child's daily fare, and let y stand for an adult's daily fare. We will use graphs to analyze this situation.

1. If only one child and one adult joined the vanpool, there is more than one possible pair of values for x and y, for example a fare of \$2 for the child, and \$10 for the adult.
   a. List three other possible (x,y) pairs.

   b. Plot these (x,y) pairs on the coordinate axes below.
   c. Make a graph that includes all possible (x,y) pairs.



2. Repeat Problem 1 assuming that one child and two adults join the vanpool. (They still need to get $12 total in fares.)

3. Repeat problem 1 assuming that two adults and three children join the vanpool.

4. a. Write equations for the graphs you drew in the previous three problems.

   b. For each equation, interpret the coefficients of x and y and the constant term in terms of the situation.

   c. Find the x-intercept and y-intercept on each of your graphs and interpret them in terms of the situation.

Negotiations

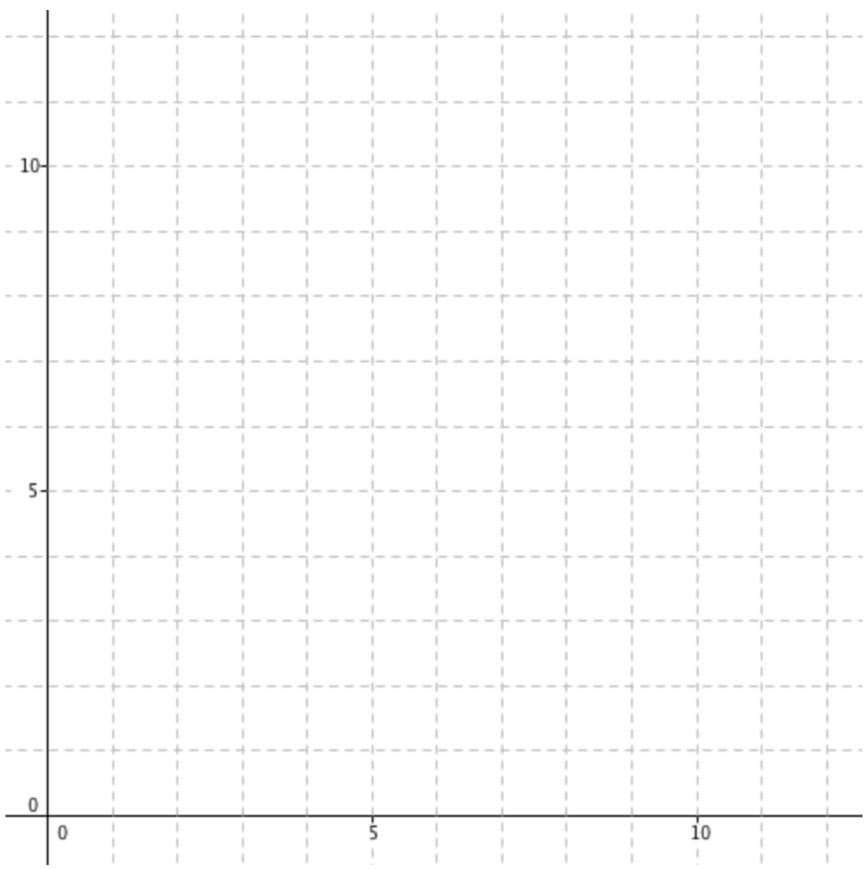
In this section, assume that the vanpool has four children and three adults.

5. a. Find possible (x,y) pairs representing the fare for children and adults.

   b. Draw a graph that shows all possible (x,y) pairs using the axes below.

   c. Label and interpret the x-intercept and y-intercept on your graph.

   d. Write an equation for the graph.



The members of the vanpool discussed how to divide the cost among them. They discussed several possible plans. In each case described in problems 6-9, do the following:

a. Figure out what the daily fares for adults and children would be. Show your work.
b. Plot a point on the graph from Problem 5 to represent your solution.

6. Frances suggested that adults pay twice as much as children because they have more money.

7. Juan thought that adults should pay $1 more than children.

8. Joanna argued that there was no reason to have different fares, since an adult and a child each occupy one seat.

9. Allan thought it was unfair to have adults pay more than children, since adults take turns driving the van. He argued that children should pay twice as much as adults.

3. Graphing *Ax + By = C*

In an earlier lesson, we saw these equations: $5x + 25y = 500$ and $x + y = 56$. In this lesson, we will learn about the graphs of such equations.

The general formula is $Ax + By = C$.
A, B, and C are called the *parameters;* x and y are the variables.

For each equation:

a. Find the parameters A, B, and C.
b. Find the x-intercept and y-intercept of the graph of the equation.
c. Graph the line by first plotting the intercepts.

1. $3x + 2y = -12$

2. $3x - 2y = 6$

3. $x + y = 6$

4. $x - y = 6$

5. $-3x + 4y = 12$

6. **Generalization**
   a. Explain how to find the x-intercept and y-intercept of the line whose equation is $Ax + By = C$.

   b. A fast way to graph a line is by finding and plotting the intercepts. Show how to use this technique to graph a line of the form $Ax + By = C$. (Choose specific values for A, B, and C.)

In the next two problems, you will explore what happens when one of the parameters is 0.

7. Graph these lines:
   a. $2y = 6$
   b. $3x = 6$
   c. $3x + 2y = 0$

8. What does the graph of $Ax + By = C$ look like if
   a. $A = 0$?

   b. $B = 0$?

   c. $C = 0$?

Finding the intercepts and connecting them with a line will work to graph many linear equations, even if they are not written in the $Ax + By = C$ format.

9. Graph these equations:
   a. $x = -2y + 10$
   b. $-2y = 4x + 8$
   c. $-2y + 10 = 2x + 8$

10. Graph these three equations on the same pair of axes. Describe what you observe.
    a. $x + 3y = 9$

    b. $2x + 6y = 18$

    c. $x + 3y = 10$

    d. Which two equations are equivalent? How does it show up on the graph?

11. **Challenge**
    a. Write the equation of a line that has x-intercept (-6,0) and y-intercept (0,4).

    b. Show how to find the equation of a line with intercepts $(p,0)$ and $(0,q)$.

4. Two Unknowns

Here is an equation with two unknowns: $y + 4x = 12$.

1. Find some values of x and y that satisfy the equation. (How many possible values are there?)

One of the (x,y) pairs satisfying this equation also satisfies the constraint that y is twice x. Written as an equation, this means $y = 2x$.

If y is twice x, then we can write $2x$ instead of y in the equation. If we do that, we get:

$$2x + 4x = 12.$$

You know how to find an x value that will satisfy this equation – in other words, to *solve* it.

2. a. Explain why the solution is x = 2.

   b. Find the (x,y) pair that satisfies $y + 4x = 12$ with the additional constraint that y is twice x.

Each of problems #3-6 includes an equation with two unknowns, and an extra constraint. Find the (x,y) pair that satisfies both the equation and the constraint.

3. $4x - 7 = y + 3$
   Constraint: y is two more than x

4. $2y + x = 5$
   Constraint: x is six less than y

5. $2x + y = 9$
   Constraint: x is three more than y

6. $2y + x = 4$
   Constraint: x and y add up to six

For each problem you just solved, the constraint could have been written as an equation. For example, the constraint that the sum of x and y is six can be written $x + y = 6$. This means that in each of problems 3-6, you found an *(x, y)* pair that satisfied two equations at the same time. We say that you *solved a system of simultaneous equations*.

Solve these systems:

7. $\begin{cases} 2x - y = 2 \\ y = 3x \end{cases}$

8. $\begin{cases} 4x + y = 10 \\ y = 6x - 20 \end{cases}$

9. $\begin{cases} x - 4y = 23 \\ x = -5y - 4 \end{cases}$

10. $\begin{cases} 3y + 2x = 7 \\ 3y = 4x - 5 \end{cases}$

5. Mind Reading

What numbers am I thinking of?

1. Their sum is 18. The second is twice as large as the first.

2. Their sum is 7. Their difference is 3.

3. The first minus the second is 3. Twice the first minus twice the second is 6.

4. One of the three problems above has more than one answer. How many answers does it have? Why?

5. Fill out the rest of this table. Throughout the table, x and y are chosen so that $2x + 3y = 16$. For the bottom row, choose your own numbers, but make sure that they satisfy the equation.

x	y	$2x + 3y$	$x + y$	$x - y$	$4x + 6y$	$x + 1.5y$
-1	6	16				
2	4	16				
	5	16				
	-6	16				
-4		16				
		16				

6. Study the table you made. In which columns are all the values the same? Why?

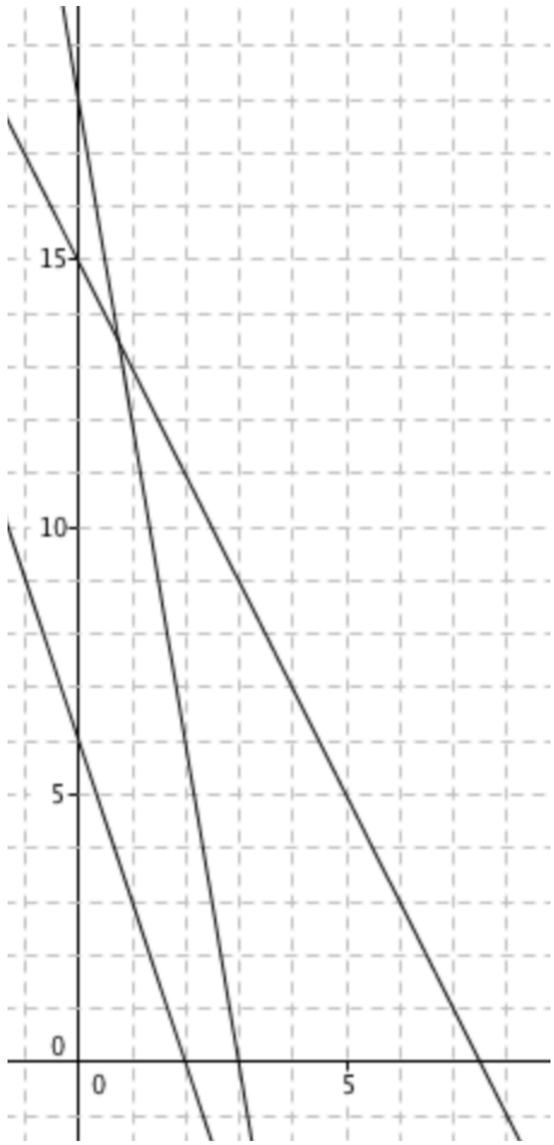
7. A crate contains two small containers and three large containers. The total weight of the crate is 16 pounds.
   a. What are some possible weights of the small and large containers? How many possibilities are there?

   b. Find the weight of four small containers and six large containers.

   c. **Challenge:** Two containers are removed from the crate, and it is weighed again. Now the crate weighs 10 pounds. Using this additional information, find possible weights for the small container and the large container. Comment on your answers.

6A. Line Intersections

In this lesson, we will learn how graphs can help us understand systems of linear equations.



This figure shows the graphs of three equations:

$3x + y = 6$ $6x + y = 18$ $2x + y = 15$

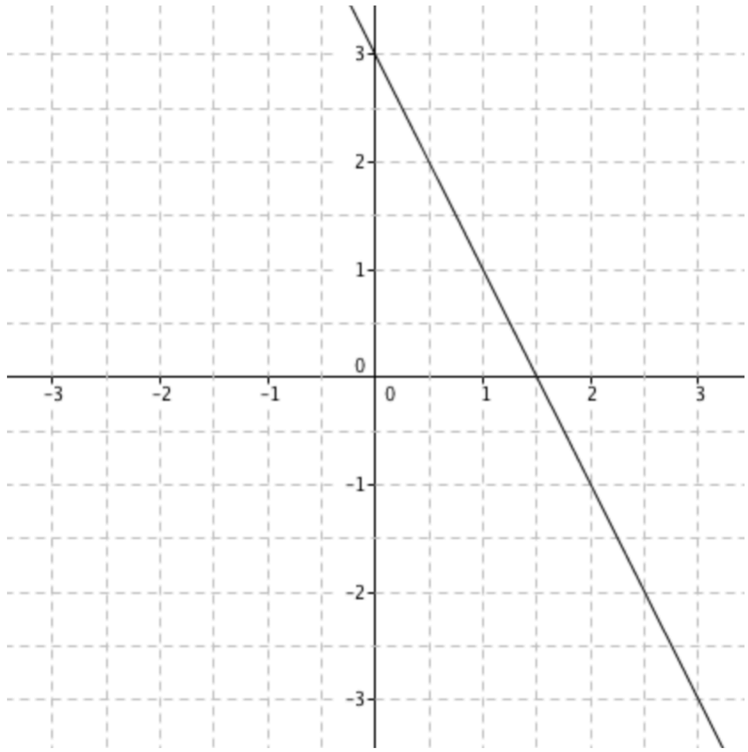
1. Label each graph with its equation.

2. There is a point on each of the lines in where the y-value is three times the x-value.
   a. Find and label these points.
   b. They should all lie on one straight line. What is the equation of this line? Draw the line.

3. Use the graph to solve the system $\begin{cases} 3x + y = 6 \\ y = 3x \end{cases}$

4. Write and solve two other systems that can be solved using the graph above.

6B. Line Intersections (continued)



This is the graph of $4x + 2y = 6$.

1. Mark and label a point on the line for which
   a. the y-coordinate is four times the x-coordinate;
   b. y is twice x;
   c. x is three less than y;
   d. y is three less than x.

2. Graph these lines on the same axes:
   a. $y = 4x$
   b. $y = 2x$
   c. $x = y - 3$
   d. $y = x - 3$

3. Write and solve four systems of linear equations that can be solved with the help of these graphs.

7. Lines and Systems

In this lesson, you will learn how graphs can help solve and understand systems of linear equations.

Lines

1. Graph these two equations on the same pair of axes. Describe what you observe.
   a. $x + 3y = 9$

   b. $x + 3y = 10$

   c. Is it possible for $x + 3y$ to be equal to 9 and 10 at the same time?

   d. How does this show up on the graph?

2. Graph the line $2x - 3y = 6$. Then write an equation that has
   a. the same graph

   b. a parallel graph

For each pair of equations in #3-5, tell whether the two graphs will be

a. the same graph;
b. parallel graphs;
c. intersecting graphs.

3. $2x + 9 = y$ and $-4x - 18 = -2y$

4. $x - y = 7$ and $x + y = 7$

5. $x + y = 7$ and $x + y = 9$

Systems

6. Solve the system: $\begin{cases} 4x - 3y = 0 \\ 2x + 3y = 18 \end{cases}$

The (x,y) pair you found satisfies both equations. Therefore the point with those coordinates should be on both graphs.

7. Graph the two lines, and verify that the point you found in #1 is on both of them.

You can use graphs to solve systems of linear equations: just graph both lines and see where they meet. The coordinates of that point satisfy both equations.

8. Graph both lines to solve this system: $\begin{cases} x + 2y = 4 \\ -3x + y = 9 \end{cases}$

9. Check that the solution you find really works by substituting x and y in the equations.

8. How Many Solutions?

Some systems have one solution, some have an infinite number of solutions, and some have none.

One or more of the following systems consists of equivalent equations, which share the same graph. One or more yields parallel lines for the graphs. The graphs for one or more of the others intersect in one point. For each system, answer these questions:

a. How many solutions does it have? How do you know?
b. If there is a unique solution, what is it?

1. $\begin{cases} x+2y=4 \\ 3x+6y=12 \end{cases}$

2. $\begin{cases} x+2y=4 \\ x+2y=8 \end{cases}$

3. $\begin{cases} x+2y=4 \\ 3x+6y=18 \end{cases}$

4. $\begin{cases} -x+2y=8 \\ x+2y=8 \end{cases}$

Graphing is a good way to see what happens when solving systems, but it is not convenient or accurate if you are graphing by hand.

9A. Combining Equations

In this lesson, you will study the relationship between solving a system of linear equations and the corresponding graphs.

1. a. Graph these two equations on the same pair of axes.

$$3x + y = 7$$
$$-2x + y = -8$$

   b. Label the point of intersection.

   c. Add these two equations to get a third equation. Graph it on the same pair of axes. What do you notice?

2. a. Graph these two equations on the same pair of axes.

$$5x - 2y = 3 \quad \text{(A)}$$
$$2x + y = 3 \quad \text{(B)}$$

   b. Label the point of intersection.

   c. Get a third equation by adding (A) + (B) + (B)

   d. Graph this equation on the same pair of axes. What do you notice?

   e. What is the solution to the following system?.

$$\begin{cases} 5x - 2y = 3 \\ 2x + y = 3 \end{cases}$$

3. Here are two equations of lines.

$$2x + 3y = 5$$
$$x + 2y = 4$$

   a. Use addition or subtraction to get the equation of a *horizontal* line that passes through their intersection.

   b. What is the solution to the following system?.

$$\begin{cases} 2x + 3y = 5 \\ x + 2y = 4 \end{cases}$$

4. Explain how "adding lines" to get horizontal and vertical lines is related to solving systems of equations.

5. Add or subtract these equations until you have a horizontal line and a vertical line through their intersection:

$$\begin{cases} 3x + 2y = 12 \\ x - 2y = -4 \end{cases}$$

9B. Combining Equations (continued)

Sometimes, you can solve a system of equations by combining the equations. Here is an example:

$$\begin{cases} x+2y=11 \\ x-2y=3 \end{cases}$$

Since we can add the same amount to both sides of the first equation, let's add $x - 2y$ on the left, and 3 on the right.

1. Explain why that is correct.

Now we have:

$$\begin{cases} 2x=14 \\ x-2y=3 \end{cases}$$

2. What happened to the y's in the first equation?

3. Solve the system, and check your answer is correct. Remember that you need values for both x and y, and that they must satisfy the original equations.

Solving this system was easier than solving most systems, since when you added one equation to the other there were no *y*'s left. The next example is more difficult.

$$\begin{cases} 2y - 6x = 16 & \text{(A)} \\ 4x + y = 1 & \text{(B)} \end{cases}$$

4. One way to solve this system is to get an equation equivalent to equation (B) by multiplying both sides by -2. Do it, and call the resulting equation (C).

5. Add equations (A) and (C) and solve the system. Remember that you need values for both x and y, and that they must satisfy the original equations.

Sometimes, you must use equivalent equations for both equations, as in this example:

$$\begin{cases} 3x + 5y = 17 & \text{(A)} \\ 2x + 3y = 11 & \text{(B)} \end{cases}$$

6. One way to solve this system is to multiply equation (A) by 2, and equation (B) by -3.
   a. Explain why this is a good idea.

   b. Do it and solve the system. Remember that you need values for both x and y, and that they must satisfy the original equations.

10. Using Systems

Celsius-Fahrenheit Conversion

Outside the United States, temperatures are measured in degrees Celsius. Scientists in the US also use degrees Celsius.

Water freezes at 0° Celsius, which is 32° Fahrenheit.
Water boils at 100° Celsius, which is 212° Fahrenheit.

1. A temperature reading can be converted from Fahrenheit to Celsius by using the formula
$$C = mF + b$$
Find m and b by using the fact that C = 0 when F = 32 and C = 100 when F = 212.

2. Find a formula for converting Celsius to Fahrenheit.

3. When the temperature increases by one degree on the Celsius scale, how much does it increase on the Fahrenheit scale? Where does this number appear in the conversion formula? Explain.

Bikes and Trikes

Julia counted 41 wheels in the preschool yard. All of them were on bikes and trikes. (She did not count training wheels.)

4. Make a table showing some possible numbers of bikes and trikes.

5. Jana counted a total of 16 bikes and trikes in the same yard. How many of each kind were there?

Legs

6. Jeanie saw some cows and chickens. She had nothing to do, so she counted their legs and heads, over and over. Here are her results:
   The first time: 93 legs, 31 heads
   The second time: 66 legs, 16 heads
   The third time: 82 legs, 29 heads
   She counted accurately only one time. How many cows and how many chickens were there? Explain.

7. Juan saw some three-legged stools and four-legged chairs. He was bored, so he counted their legs. There were 59 legs. Then he put six pennies on each stool, and eight nickels on each chair. (He thought it would make a good math problem.)
   a. He used 118 coins. Can you tell how many of chairs and stools there were? Explain.

   b. The total value of the coins was $3.74. Can you tell how many chairs and stools there were? Explain.

   c. How many of each kind of coin did he use?

11. Systematic Practice

Solve these systems. Some have one (x,y) solution, some have an infinite number, some have none.

1. $\begin{cases} 5y - 4x = -9 \\ 5y = 3x - 7 \end{cases}$

2. $\begin{cases} 5x + 3y = -15 \\ y = 2x + 6 \end{cases}$

3. $\begin{cases} 5x - 3y = -29 \\ x = 2 - 2y \end{cases}$

4. $\begin{cases} 2x + 3y = 9 \\ 4x = 6 - 2y \end{cases}$

5. $\begin{cases} 4x - y = 5 \\ 3y = 6x + 3 \end{cases}$

6. $\begin{cases} 6x - 2y = -16 \\ 4x + y = 1 \end{cases}$

7. $\begin{cases} 5x + 7y = 1 \\ x + 7 = 1 \end{cases}$

8. $\begin{cases} 3 - x = 4y \\ x = -2y - 9 \end{cases}$

9. $\begin{cases} 8x - 4y = 0 \\ 2x = y \end{cases}$

10. $\begin{cases} y = 4 + x \\ y = 7x + 10 \end{cases}$

11. $\begin{cases} 4x - y = 2 \\ y = 4x + 1 \end{cases}$

12. $\begin{cases} 6x - 2y = -16 \\ 4x + y = 1 \end{cases}$

13. $\begin{cases} 2x - 3y = 7 \\ 3x - 4y = 15 \end{cases}$

14. $\begin{cases} x = 6 + 3y \\ 3y = 3 + x \end{cases}$

15. $\begin{cases} y - 12 = 4x \\ 2y - 8x = 24 \end{cases}$

16. $\begin{cases} y = 42 - 4x \\ 6x = 50 + 5y \end{cases}$

17. $\begin{cases} y - 12 = 4x \\ 2y = 8x + 24 \end{cases}$

18. $\begin{cases} 2y - 2x = 7 \\ y - x = 3.5 \end{cases}$

19. Which of these problems has one solution? Which has an infinite number of solutions? Which has no solution? Explain.

a. I'm thinking of two numbers. Their sum is 10. Twice the first plus twice the second is 20.

b. I'm thinking of two numbers. Their sum is 6. Their difference is 10.

c. I'm thinking of two numbers. The second is 5 more than the first. The second minus the first is 6.