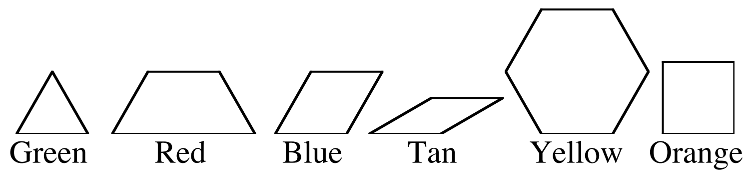
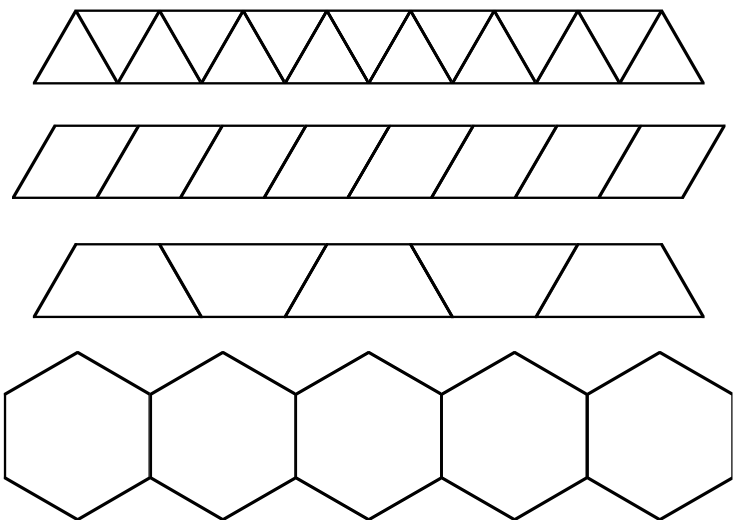
Pattern Block Trains

This activity requires pattern blocks, which are available from many vendors of math educational products. #1-5 can be adapted for students in grades 5 and up, but this version is intended for teachers.

The pattern blocks:



1. Build pattern block trains by connecting blocks *edge-to-edge,* from left to right.

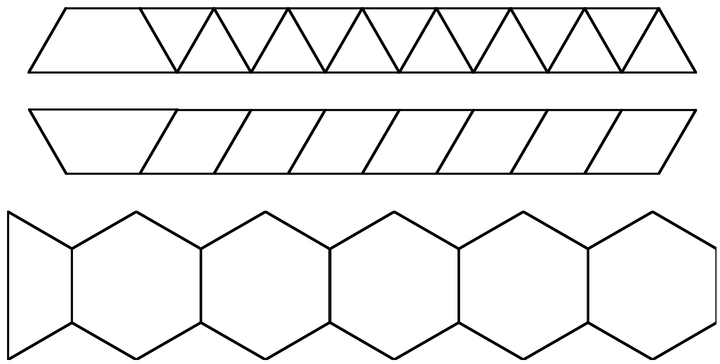


What is the perimeter of each train, as a function of the number of blocks? For example, for the blue train, P = 2n + 2, where P is the perimeter, and n is the number of blocks.

2. The formulas are in the form P = mn + b. Explain the meaning of the m and the b in terms of the blocks.

Fancy Trains

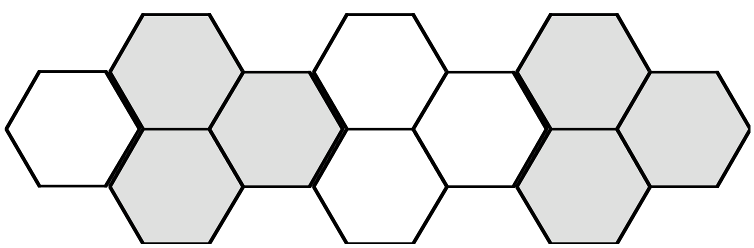
These trains start with a red block, and continue in the same way as above.



3. What is the perimeter of each train, as a function of the number of blocks? Do not count the initial red block as part of n, but include its contribution to the perimeter. For example, for the blue train, P = 2n + 5.

4. Build other trains whose perimeters have a different value for b than 2 or 5.

Now consider the following train. In order to get a linear function, we only look at the perimeter after adding three blocks: for n=1, P=6; for n=4, P=14; for n=7, P=22; for n=10, P=30; and so on.



5. What is the formula for this train?

The Big Question

6. What values of m are possible for pattern block trains?

Pattern Block Trains: Teachers' Notes

With younger children, do not start by asking for a formula. The sequence should be to make a table, then to describe in words how to get the perimeter from the number of blocks, and finally, if appropriate, to get a formula.

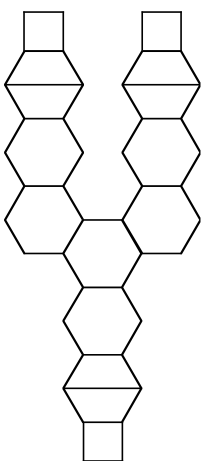
In middle school, it's not uncommon for students to come up with equivalent formulas (e.g. some students get P=2n+2 and others get P=2(n+1). This is a great opportunity to discuss the distributive law, and the importance of symbol manipulation to facilitate communication.

In beginning algebra, it is a good idea to graph the functions, and discuss the interpretation of slope and intercept in the formulas.

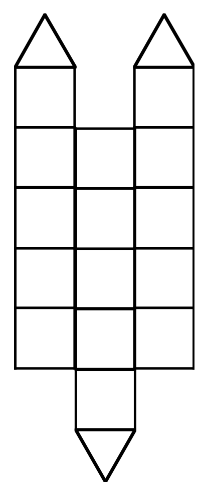
For #5, once an order of laying down the blocks is agreed upon, it is interesting to graph all the (n,P) pairs. This should yield three parallel graphs: one for n = 1, 4, 7, 10, … one for n = 2, 5, 8, 11, … and one for n = 3, 6, 9, 12, … Similarly for trains where you alternate blocks (orange, yellow, orange, yellow, etc…) and other complex trains.

#6 can be explored as an open-ended question. It is unlikely any student can fully solve it, let alone prove the solution, but it provides a great arena for discovery.

Ted Slaman of the UC Berkeley Math Department asked question #6, and answered it by proving that m can take any rational value with $0 < m \leq 4$. I will not include the whole proof here, but it is based on showing how to find a train for any positive rational number P/n less than 4. This can be done with variations on one or the other of these two trains:



Train 1: Each "wagon" consists of a number of hexagons, and ends with one of the pattern blocks. Any hexagon can be replaced by two, three, four, five, or six blocks, without changing the perimeter. In the case shown here, m = 14/5 (5 blocks contribute 14 units of perimeter.) Variations on this can yield any rational m from 2/3 to 4.



Train 2: Each "wagon" consists of a number of squares, and may end with a triangle. The stacks can be staggered as necessary. In the case shown here, m = 5/6 (6 blocks contribute 5 units of perimeter.) Note that the numerator is always at least 2, but that does not prevent us from dealing with fractions of the type 1/n, which can be done as 2/2n. Variations on this can yield any positive rational m less than 2.

Of course, other approaches are possible.