Reimagining High School Math

more access, more challenge

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Is there such a thing as pedagogy for high school math?

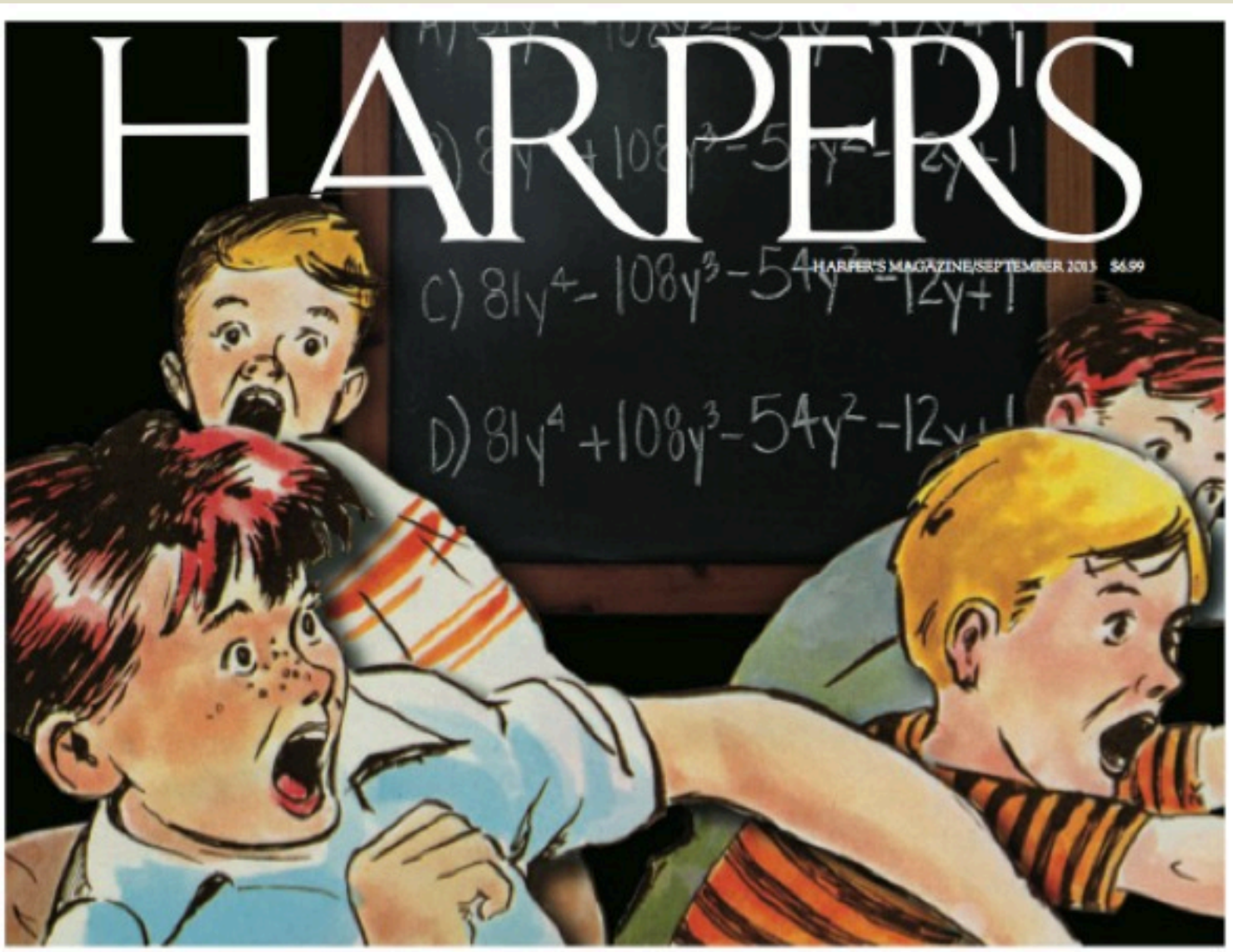
Most math classes,
most of the time,
look just like they did
10, 20, or 30 years ago

Most math classes
look the same
at very different schools

Perhaps there's
only one way
to teach math!

Does it work?

Yes, for a few



WRONG ANSWER

The Case Against Algebra II

By Nicholson Baker

- We reward blind obedience
- Many students hate this!
- What they “learn” under these conditions cannot stick

It does not have to be this way

Cooperative Learning

Dynamic Geometry

Manipulatives

Graphing Calculator

Student Presentations

Writing

Class Discussion

Reports

Using the Web

Problem of the Week

Computer Algebra Systems

Drills

Labs

Making a Poster

Socratic Questioning

“Real World” Applications

Projects

Guided Inquiry

Direct Instruction

etc.

How can you do these things when...

- ◊ trying to meet standards
- ◊ preparing kids for SATs and APs
- ◊ worrying about college preparation
- ◊ navigating parental pressures
- ◊ dealing with colleagues who may not agree

Will technology save us?

No

...but it does have implications

Speed and accuracy are no longer legitimate priorities for math education

We can no longer divorce skills
from understanding,
nor can we consider obsolete skills
to be foundational

111 ÷ 13

Adding:
13 + 13 + 13 + ...

Subtracting:
111 − 13 − 13 − 13 − ...

Multiplying:
13 × 10 is too big
13 × 8

Most of the time, we are prisoners of tradition and the textbook

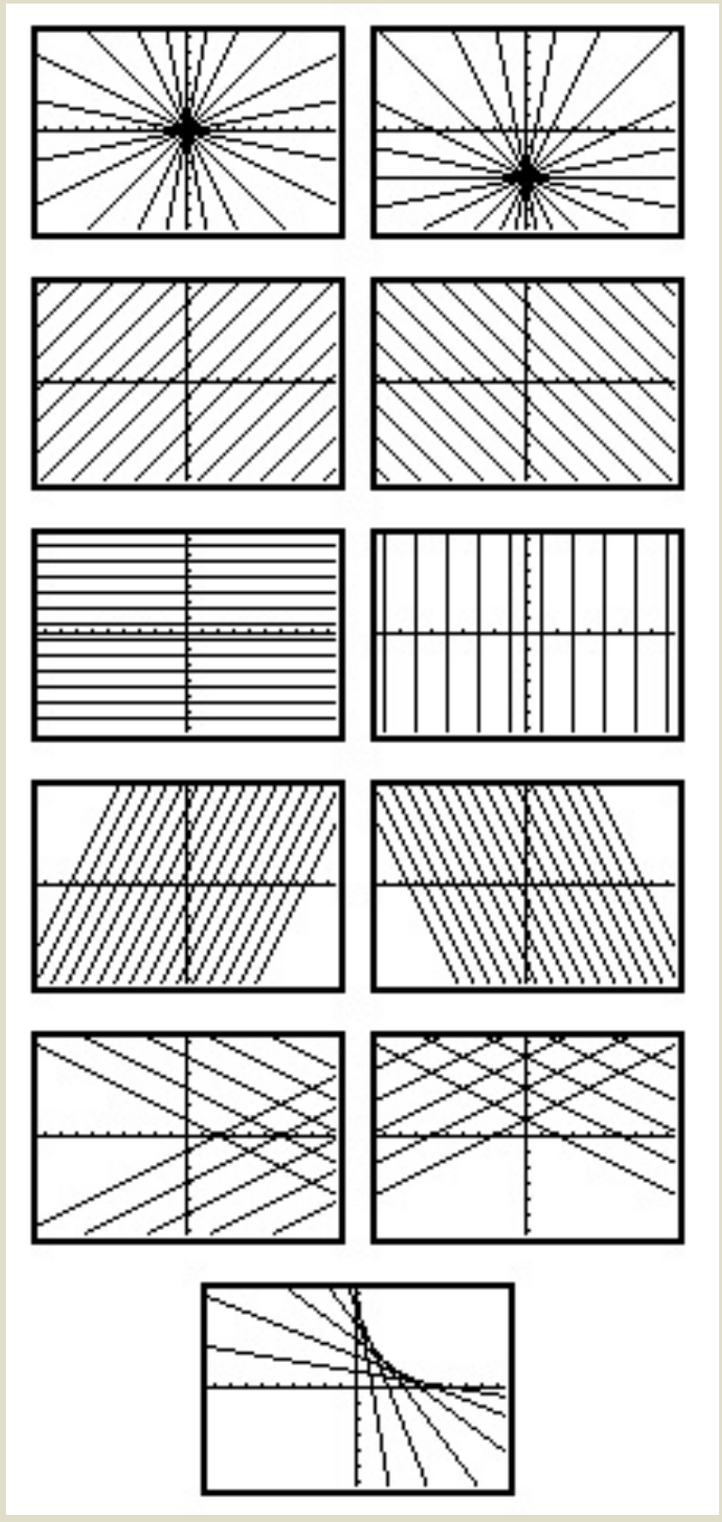
Escape!

Profound change is possible:
start now

Use learning tools

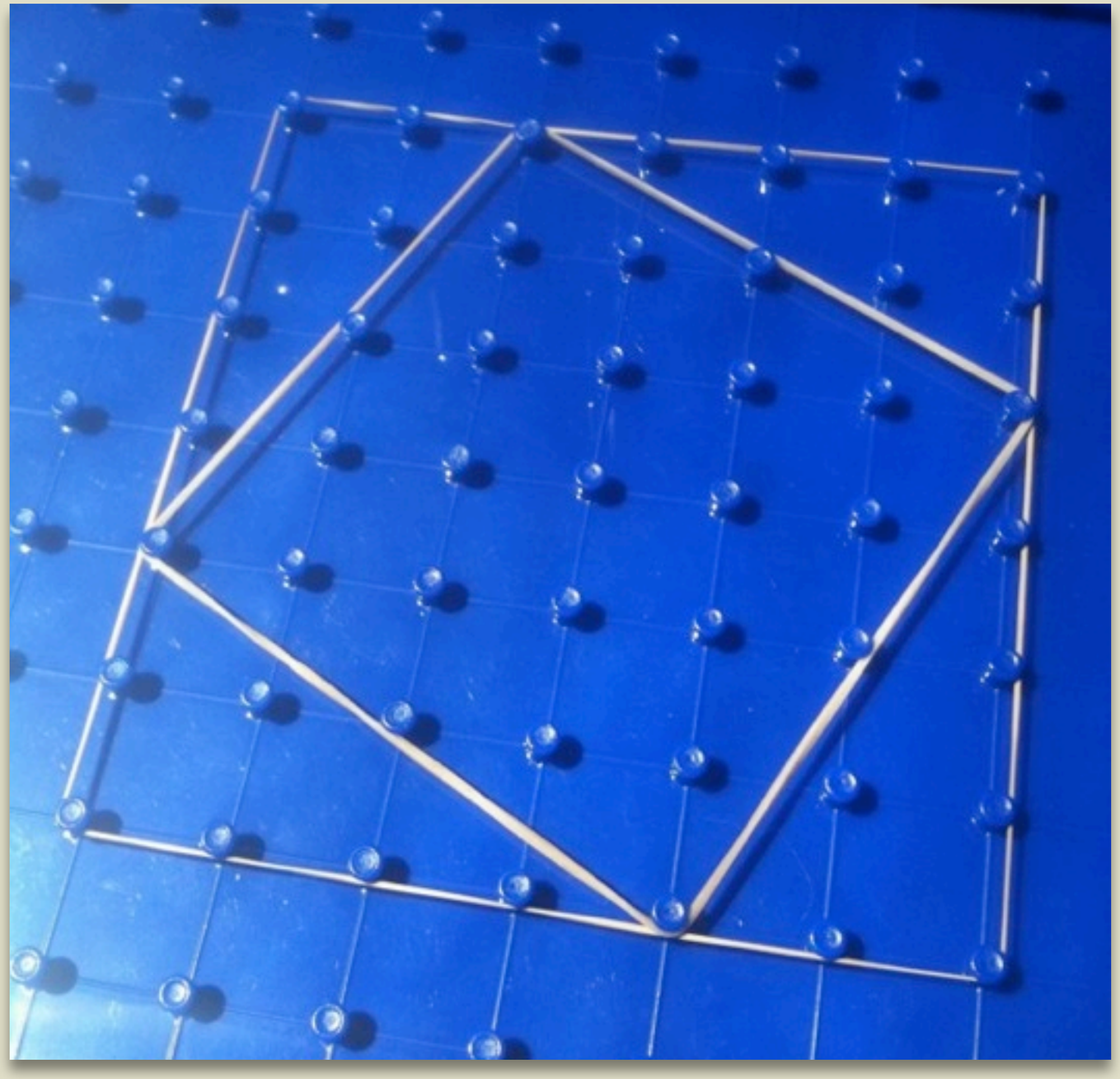
Graphing calculator

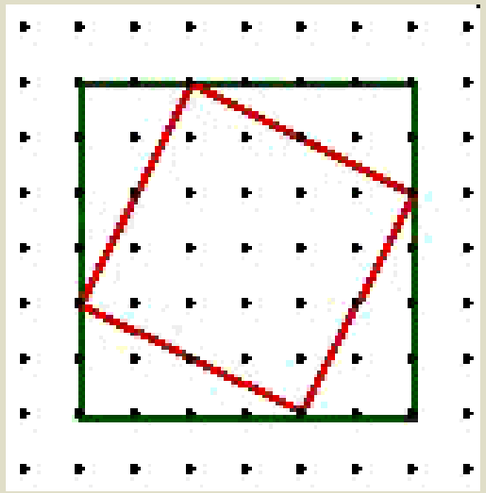
Make These Designs



Geoboard

Make a Square





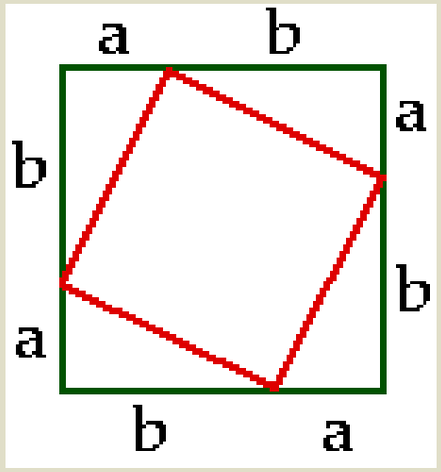
What is the area of the red square?

outer square: 6^2

each triangle: $\frac{2 \cdot 4}{2} = 4$

inner square: $36 - 4 \cdot 4 = 20$

Generalize

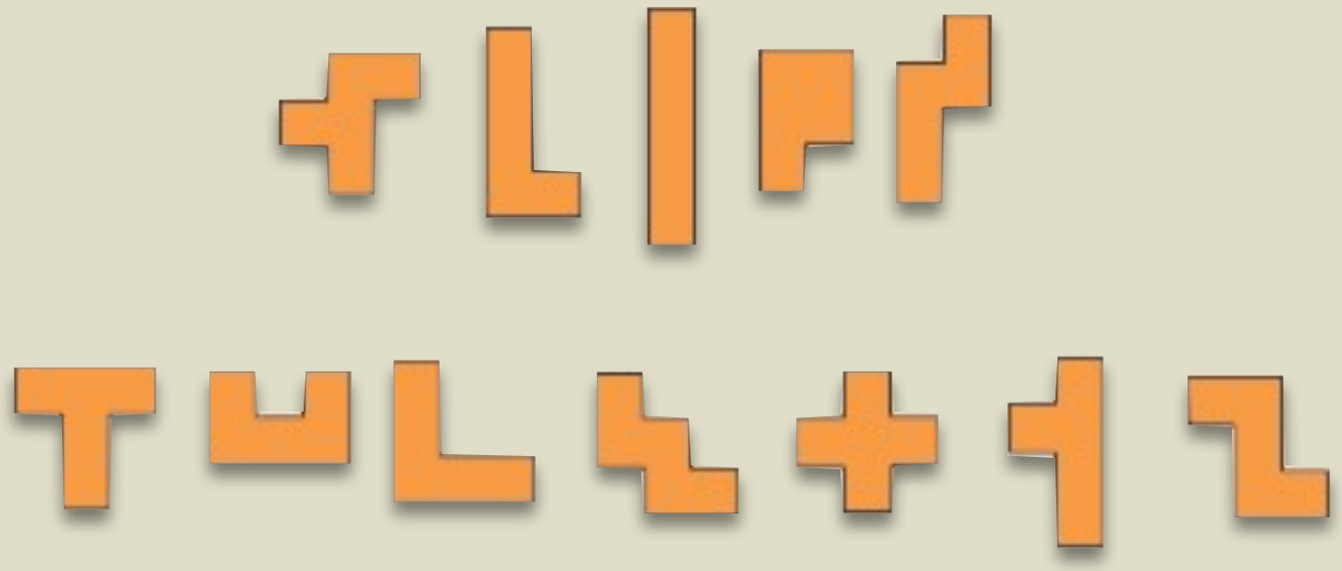


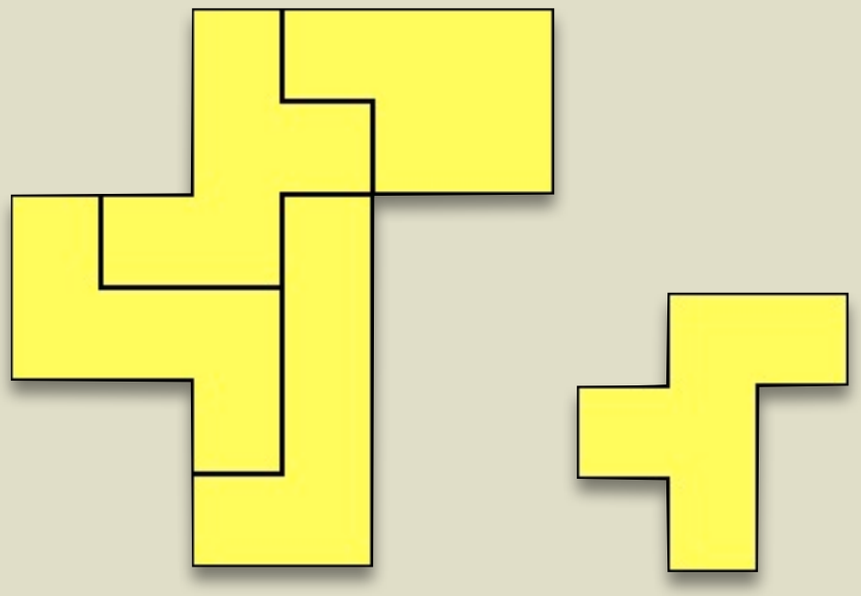
outer square: $(a + b)^2$

each triangle: $\frac{a \cdot b}{2}$

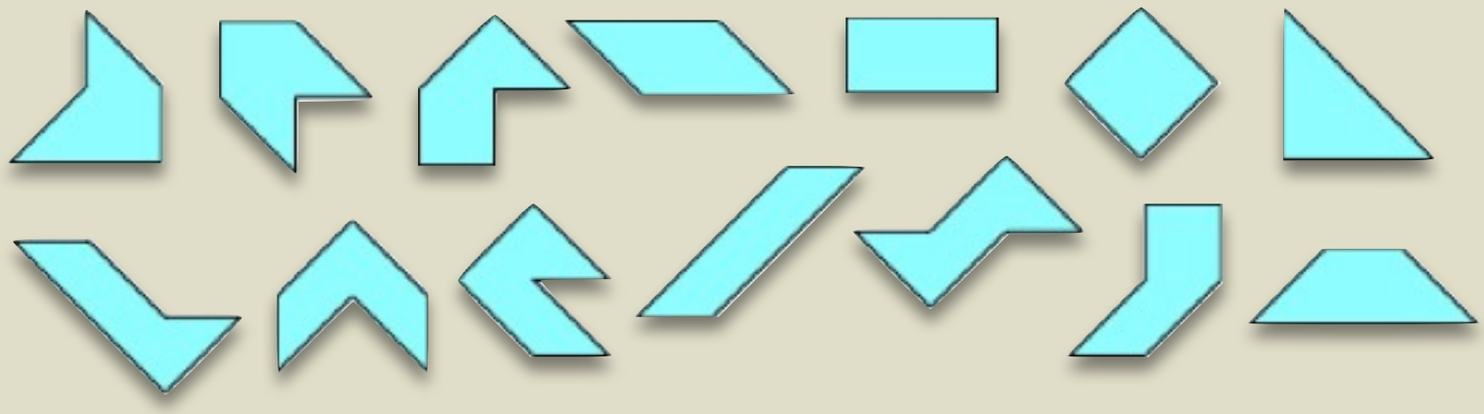
inner square: $(a + b)^2 - 2ab = a^2 + b^2$

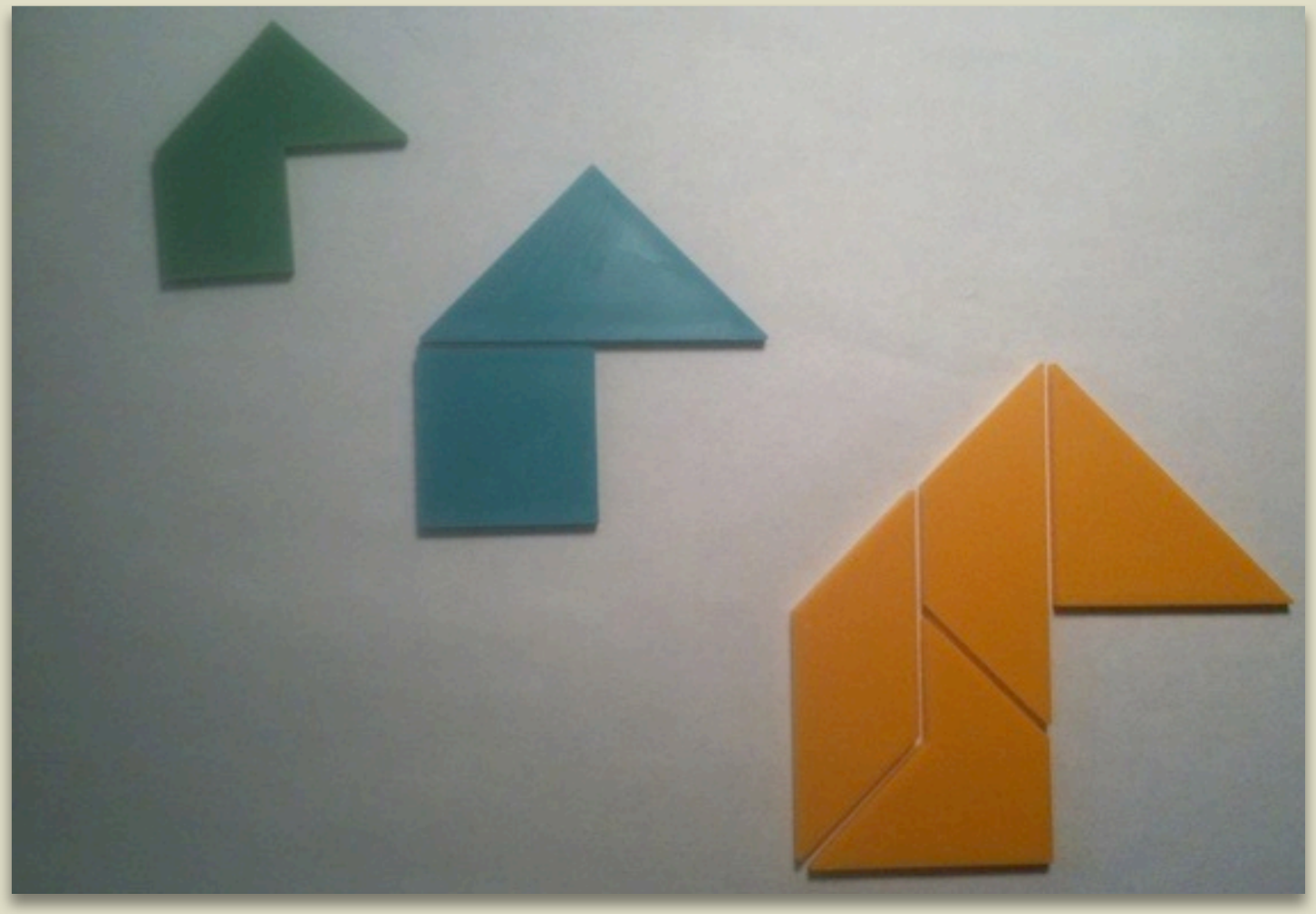
Puzzle



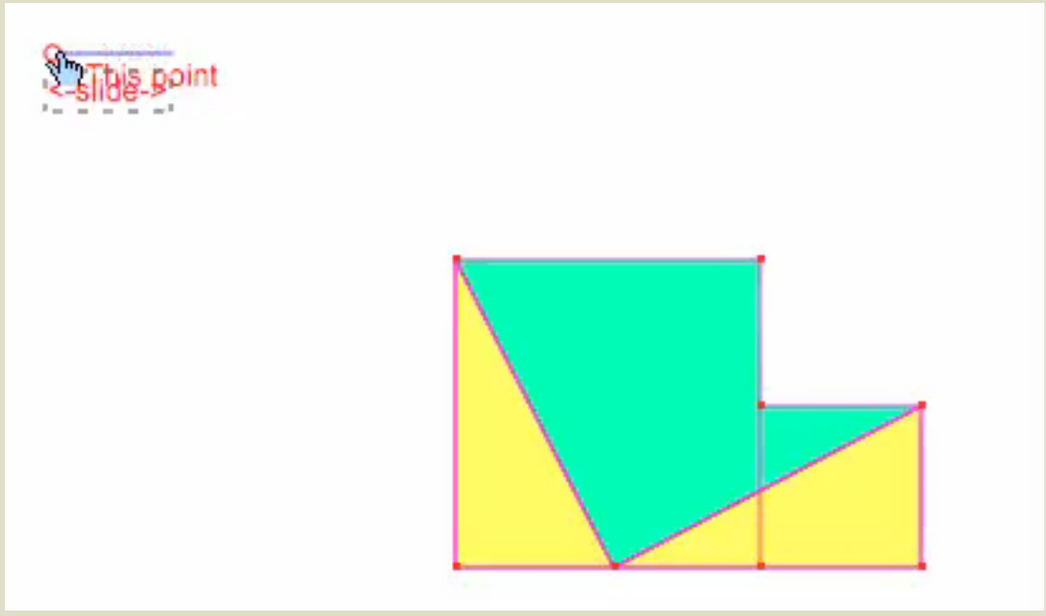


Puzzle





Explain



Use multiple representations

if $0 = ax^2 + bx + c$

then complete the square...

$$0 = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$-\frac{c}{a} = x^2 + \frac{b}{a}x$$

$$-\frac{c}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$$

$$-\frac{c}{a} + \left(\frac{b^2}{4a^2}\right) = \left(x + \frac{b}{2a}\right)^2$$

$$-\frac{4ac}{4a^2} + \frac{b^2}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\sqrt{\frac{b^2 - 4ac}{4a^2}} = \sqrt{\left(x + \frac{b}{2a}\right)^2}$$

$$\pm\frac{\sqrt{b^2 - 4ac}}{2a} = x + \frac{b}{2a}$$

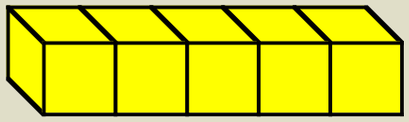
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

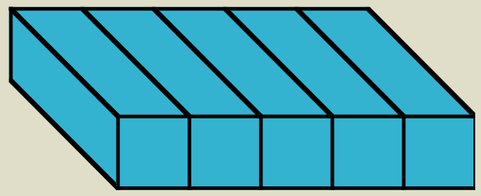
So $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$!!

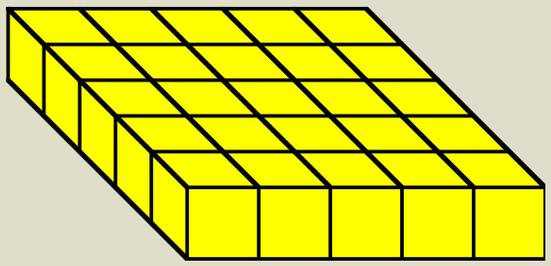
The Lab Gear

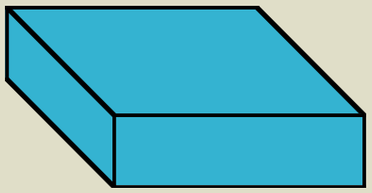
1

x

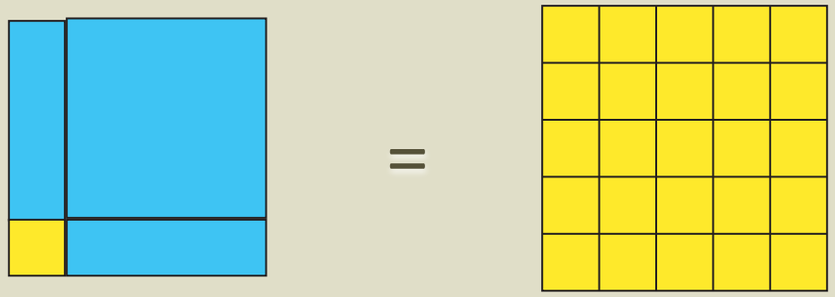
5 

5x 

25 

x ^{2} 

$$x^2 + 2x + 1 = 25$$



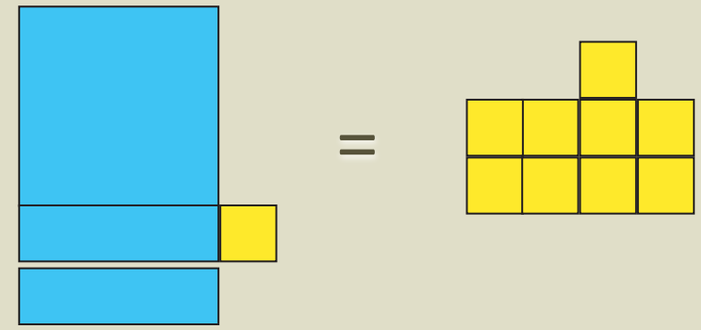
(Equal Squares)

$$x + 1 = 5$$

or

$$x + 1 = -5$$

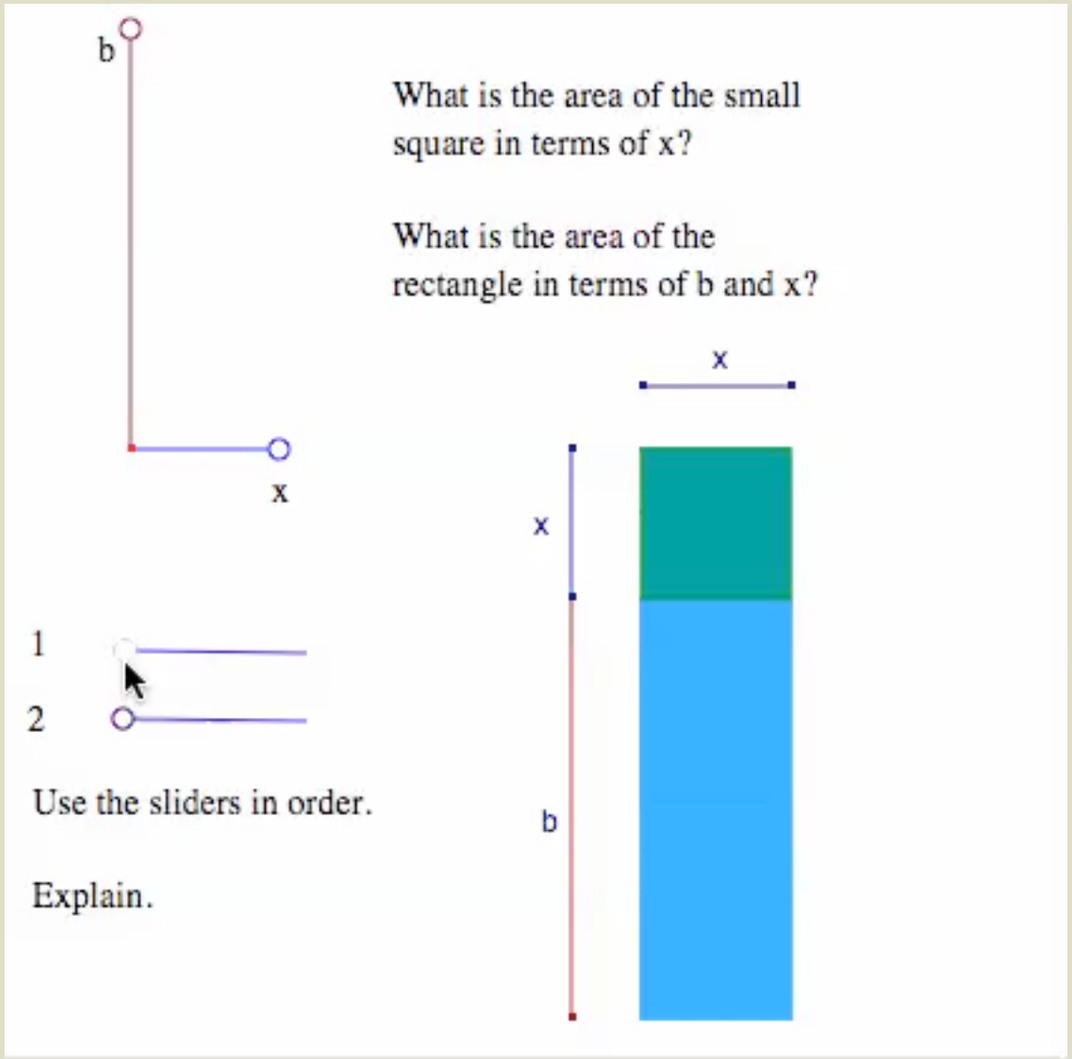
$$x^2 + 2x = 8$$



Complete the square

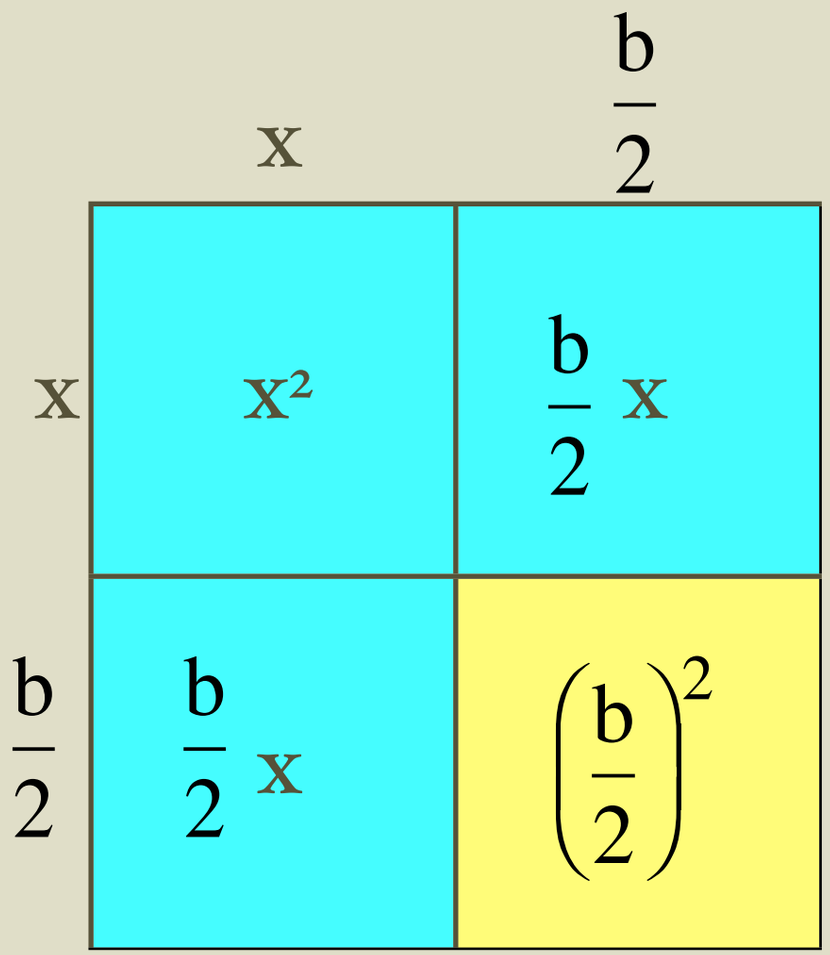
(We're back to equal squares)

Continuous version:



Symbolic Version

$$x^2 + bx + ?$$

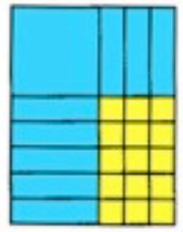


from *Lab Gear Activities for Algebra 1*, by Henri Picciotto --www.picciotto.org/math-ed

$(x + 8)x$
$= x^2 + 8x$

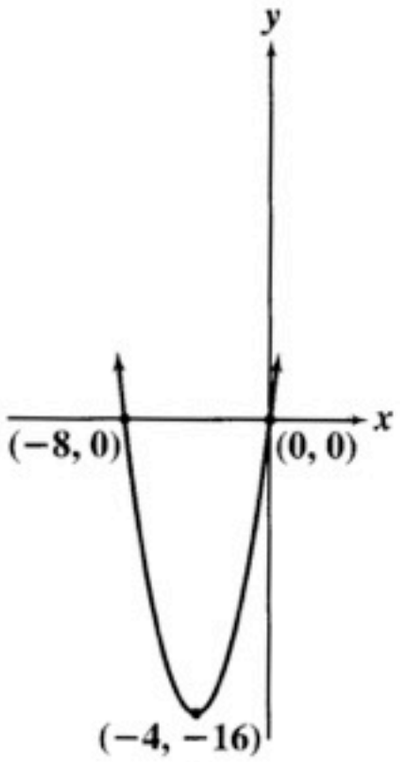
$(x + 7)(x + 1)$
$= x^2 + 8x + 7$

$(x + 6)(x + 2)$
$= x^2 + 8x + 12$

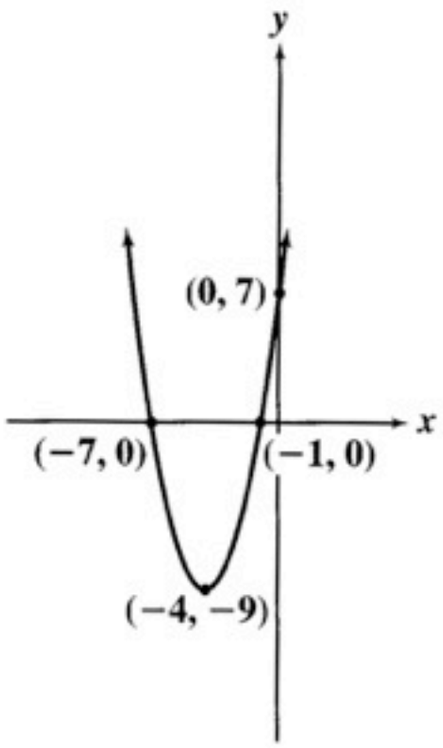


$(x + 5)(x + 3)$
$= x^2 + 8x + 15$

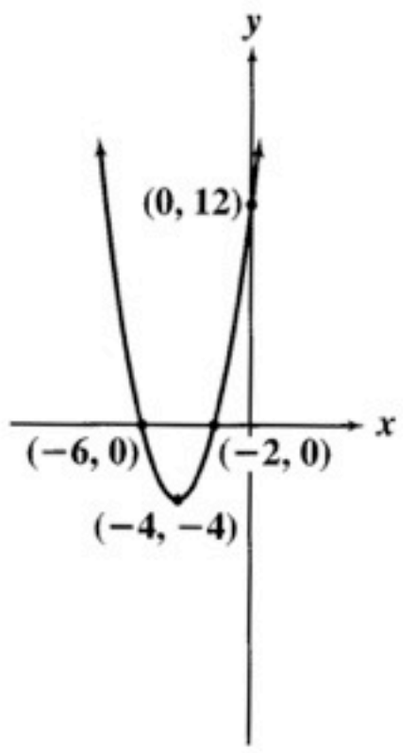
$(x + 4)^2$
$= x^2 + 8x + 16$



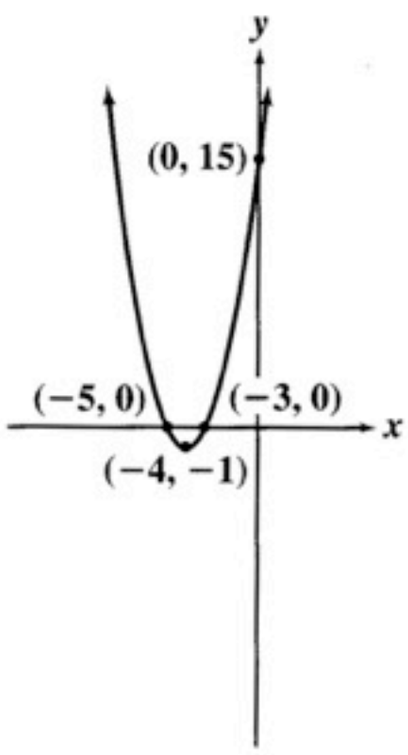
$y = x^2 + 8x$



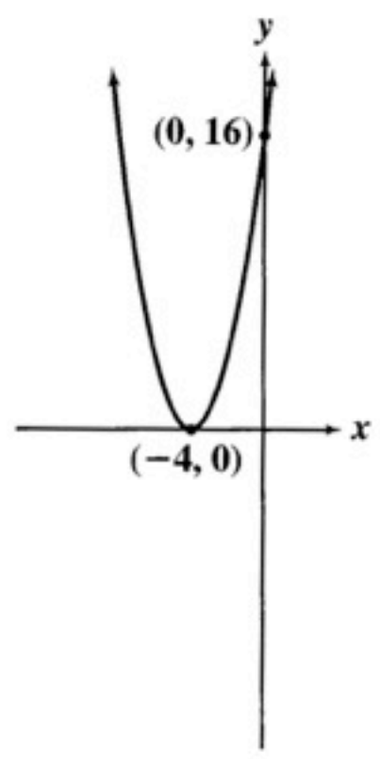
$y = x^2 + 8x + 7$



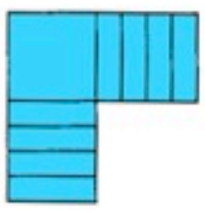
$y = x^2 + 8x + 12$

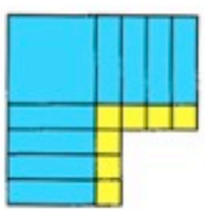


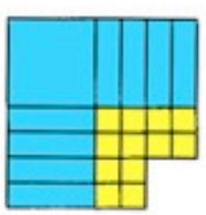
$y = x^2 + 8x + 15$

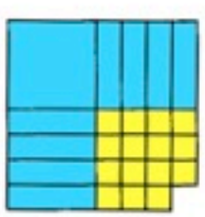


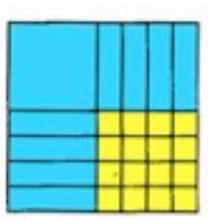
$y = x^2 + 8x + 16$











Teach for understanding

Extend exposure:
constant forward motion
eternal review

Topic 1	Week 1 class work	Week 2 homework	Week 3 quiz	Week 4 "recycle"	
Topic 2		Week 2 class work	Week 3 homework	Week 4 quiz	Week 5 "recycle"

Sequencing:

◊ do important / difficult topics early

◊ separate related topics

Understanding...

◊ is difficult to encapsulate in a checklist

◊ cannot be easily conferred by explanations

◊ is difficult to assess

◊ is not always valued by students and parents

◊ is the most important part of our job

Making changes: how?

There is no one way

Prioritize habits of mind,
not “coverage”

- ◊ Teacher collaboration is the engine
- ◊ Create a faculty culture of constant evaluation
- ◊ Core classes matter more than electives
- ◊ Each teacher eventually teaches all the core classes
- ◊ Schedule some summer work
- ◊ Go back and forth: pedagogy and curriculum
- ◊ Set priorities! / Be realistic

Guiding Questions

- Is our math program consistent with our school's mission and philosophy?
- How well does it work for different types of students?
- Is each unit age-appropriate?
- How many students take math beyond the requirements?
- What is our next step?

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